6. Ratio, Proportion and Variation

- In many situations, comparison between quantities is made by using division i.e., by observing how many times one quantity is in relation to the other quantity. This comparison is known as **ratio.** We denote it by using the symbol ':'.

- A ratio may be treated as a fraction. For example, 3:11 can be treated as $\frac{3}{11}$.

- We can compare two quantities in terms of ratio, if these quantities are in the same unit. If they are not, then they should be expressed in the same unit before the ratio is taken.

  For example, if we want to compare 70 paise and Rs 3 in terms of ratio then we have to convert Rs 3 into paise.

  Rs 3 = 300 paise

  Hence, required ratio $\frac{70}{300} = 7:30$

- Four quantities are said to be in proportion, if the ratio of first and second quantities is equal to the ratio of third and fourth quantities.

  For example, to check whether 8, 22, 12, and 33 are in proportion or not, we have to find the ratio of 8 to 22 and the ratio of 12 to 33.

  $$8:22 = \frac{8}{22} = \frac{4}{11} = 4:11$$
  $$12:33 = \frac{12}{33} = \frac{4}{11} = 4:11$$

  Therefore, 8, 22, 12, and 33 are in proportion as 8:22 and 12:33 are equal.

- When four terms are in proportion, the first and fourth terms are known as extreme terms and the second and third terms are known as middle terms.

  In the above example, 8, 22, 12, and 33 were in proportion. Therefore, 8 and 33 are known as extreme terms while 22 and 12 are known as middle terms.

- If two ratios are equal then we say that they are in proportion and use the symbol '::' or '=' to equate the two ratios.

  For example, 8:36 and 14:63 are equal as 836 = 29 and 14:63 = $\frac{14}{63} = \frac{2}{9}$

  Since 8:36 and 14:63 are in proportion, we write it as 8:36 :: 14:63 or 8:36 = 14:63.

- Two quantities, *x* and *y*, are said to be in **direct proportion**, if they increase (or decrease) together in such a manner that the ratio of their corresponding values remains constant. That is, $\frac{x}{y} = k$ where *k* is a positive number.

For example, price of wheat per kg and the weight of wheat that can be brought are in direct proportion as more the weight of wheat, more will be the cost.

- If y_1, y_2 are the values of y corresponding to the values x_1, x_2 of x respectively then $\frac{x_1}{y_1} = \frac{x_2}{y_2}$ is a case of direct proportion.
- Two variables x and y will be in direct proportion if $\frac{x}{y} = k$ or $x = ky$, where the constant k is known as constant of proportionality of the direct proportion. Thus, to check whether the variables x and y are in direct proportion, we need to find the ratio $\frac{x}{y}$ for their corresponding values. If this ratio remains constant, then the variables are in direct proportion, otherwise they are not.

- Two quantities, x and y, are said to be in **inverse proportion**, if an increase in x causes a proportional decrease in y (and vice-versa) in such a manner that the product of their corresponding values remains constant. That is, $xy = k$, where k is a positive number.
- Two variables x and y will be in inverse proportion if $xy = k$, where the constant k is known as constant of proportionality of the inverse proportion. Thus, to check whether the two variables x and y of a given situation are in inverse proportion or not, we have to calculate the product of the value of variable x with its corresponding value of the variable y. If all these products are equal, then we can say that the variables x and y are in inverse proportion, otherwise not.

  For example, $x = 1, y = 20$ and $x = 5, y = 4$ are in inverse proportion.

  Here, $1 \times 20 = 20$

  $5 \times 4 = 20$

  It can be seen that $x \times y = 20$, which is constant for both observations.

  Therefore, x and y are in inverse proportion.

- Properties related to ratio and proportion.

  (1) If $a:b::c:d$, then $b:a::d:c$

  This property is called **invertendo.**

  (2) If $a:b::c:d$, then $a:c::b:d$

  This property is called **alternendo.**

  (3) If $a:b::c:d$, then $(a + b): b:: (c + d):d$

  This property is called **componendo.**

  (4) If $a:b::c:d$, then $(a - b): b:: (c - d):d$

This property is called **dividendo.**

(5) If $a:b::c:d$, then $(a + b): (a - b) :: (c + d) : (c - d)$

This property is called **componendo and dividendo.**

(6) If $a:b::c:d$, then $a: (a - b)::c: (c - d)$

This property is called **convertendo.**

- **Theorem on equal ratios:**

If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\ldots$ and $p, q, r\ldots$ are non zero numbers such that $pb+qd+rf\ldots\neq 0$ then

$$\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\ldots=\frac{pa+qc+re+\ldots}{pb+qd+rf+\ldots}$$

In particular, we have following formula which is commonly used.

$$\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\frac{a+c+e}{b+d+f}$$

1. Order relation between ratios is used to compare the ratios.

For two ratios $p:q$ and $r:s$,

(i) if $p\times s>q\times r$, then $\frac{p}{q}>\frac{r}{s}$ i.e., $p:q>r:s$ where $q>0, s>0$

(ii) if $p\times s<q\times r$, then $\frac{p}{q}<\frac{r}{s}$ i.e., $p:q<r:s$ where $q>0, s>0$

(iii) if $p\times s=q\times r$, then $\frac{p}{q}=\frac{r}{s}$ i.e., $p:q=r:s$ where $q>0, s>0$

For example, for the ratios 2 : 3 and 196 : 201, we have

$p \times s = 2 \times 201 = 402$ and $q \times r = 3 \times 196 = 588$

i.e., $p\times s<q\times r$

$\Rightarrow 2 : 3 < 196 : 201$

- If a, b, c, $d\ldots$ are some (non-zero) quantities of the same kind then a, b, c, d … are said to be in **continued proportion,** if

$$\frac{a}{b}=\frac{b}{c}=\frac{c}{d}=\ldots$$

For example, 2, 6 and 18 are in continued proportion as $\frac{2}{6}=\frac{6}{18}$.

- The method in which we first find the value of one unit and then the value of the required number of units is known as **unitary method**.

Example:

If 15 men can do a piece of work in 10 days, then in how many days can 6 men do the same work?

Solution:

This is the case of indirect variation since more the number of men, less will be the number of days required to finish the work.

It is given that 15 men can do the work in 10 days.

∴One man can do the work in (10 × 15) days.

Hence, 6 men can do the work in $\frac{10\times15}{6}$ days = 25 days

• **Joint variation:**

If x varies as z when y is constant and x varies as y when z is constant, then x varies jointly as yz, when both y and z vary.

Mathematically, $x \propto yz$

$\Rightarrow x = kyz$ (k is a constant)

$\Rightarrow k = \frac{x}{yz}$

For example, area of rectangle is directly proportional to the product of its length and breadth *i.e.*, area of rectangle changes with the change in its length and breadth.

- **Percentages** are numerators of fractions with denominator 100. It is represented by the symbol % and means hundredths too, i.e., 25% $= \frac{25}{100}$ = 0.25
- Fractional numbers, whole numbers and decimals can be converted into percentages by multiplying them by 100%.

  Note: Percentages related to proper fractions are less than 100 whereas percentages related to improper fractions are more than 100.

  For example, $1\frac{1}{4} = \frac{5}{4} \times 100\% = 125\%$

- To convert ratio to percentage, we proceed as follows:

  Consider the ratio a:b.

  Sum of parts = $a + b$

  Percantage form = $\frac{a}{a+b} \times 100\%$

- Percentages can be converted into fractions or decimals by dividing them by 100.

  For example, 35% can be converted to decimals and fraction as follows:

$35\% = \frac{35}{100} = 0.35$

$35\% = \frac{35}{100} = \frac{7}{20}$

- To convert percentage to ratio, we have to find the ratio of the percentages of the two quantities.
- When added, all parts of a whole give whole or 100%.