SAMPLE PAPER - II

MATHEMATICS

CLASS - XII

Time : 3 Hours **Max. Marks : 100**

General Instructions

1. All questions are compulsory.
2. The question paper consist of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one mark each, section B comprises of 12 questions of four marks each and section C comprises of 07 questions of six marks each.
3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice. However, internal choice has been provided in 04 questions of four marks each and 02 questions of six marks each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculators in not permitted. You may ask for logarithmic tables, if required.

SECTION A

1. Write the number of all one-one functions from the set $A = \{a, b, c\}$ to itself.

2. Find x if $\tan^{-1} 4 + \cot^{-1} x = \frac{\pi}{2}$

3. What is the value of $|3\ I_3|$, where I_3 is the identity matrix of order 3?

4. For what value of k, the matrix $\begin{bmatrix} 2\text{-}k & 3 \\ -5 & 1 \end{bmatrix}$ is not invertible?

5. If A is a matrix of order 2x3 and B is a matrix of order 3x5, what is the order of matrix $(AB)'$ or T?

6. Write a value of $\int \frac{dx}{\sqrt{4-x^2}}$.

7. Find f(x) satisfying the following :

   $\int e^x (\sec^2 x + \tan x)dx = e^x\ f(x)+c$

8. In a triangle ABC, the sides AB and BC are represented by vectors $2\hat{i}-\hat{j}+2\hat{k}$, $\hat{i}+3\hat{j}+5\hat{k}$ respectively. Find the vector representing CA.

9. Find the value of λ for which the vector $\vec{a} = 3\hat{i}+\hat{j}-2\hat{k}$ and $\vec{b} = \hat{i}+\lambda\hat{j}-3\hat{k}$ are perpendicular to each other.

10. Find the value of λ such that the line $\frac{x-2}{9} = \frac{y-1}{\lambda} = \frac{z+3}{-6}$ is perpendicular to the plane 3x-y-2z=7

SECTION B

11. Show that the function $f : R \rightarrow R$ defined by $f(x) = 2x^3-7$, for $x \in R$ is bijective.

OR

Let f,g : $R \rightarrow R$ be defined as $f(x) = |x|$ and $g(x) = [x]$ where [x] denotes the greatest integer less than or equal to x. Find fog $\left(\frac{5}{2}\right)$ and gof $\left(-\sqrt{2}\right)$.

12. Prove that $\tan^{-1}1+\tan^{-1}2+\tan^{-1}3 = \pi$

13. If $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$, show that $A^2-5A-14I = 0$. Hence find A^{-1}.

14. Show that $f(x) = |x-3|, \forall x \in R$, is continuous but not differentiable at x=3.

OR

If $\tan\left(\frac{x^2-y^2}{x^2+y^2}\right) = a$, then prove that $\frac{dy}{dx} = \frac{y}{x}$

15. Verify Rolle's Theorem for the function f, given by $f(x) = e^x (\sin x - \cos x)$ on $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$

16. Using differentials, find the approximate value of $\sqrt{25 \cdot 2}$

OR

Two equal sides of an isoceles triangle with fixed base 'a' are decreasing at the rate of 9 cm/second. How fast is the area of the triangle decreasing when the two sides are equal to 'a'.

17. Evaluate $\int_{-1}^{\frac{1}{2}} |x \cos(\pi x)|\, dx$.

18. Solve the following differential equation :

$y e^{x/y} dx = (x e^{x/y} + y) dy$

19. Solve the following differential equation :

$(1+y+x^2y)dx + (x+x^3)dy = 0$, where y=0 when x=1

20. If $\bar{a}, \vec{b}$ and $\bar{c}$ are three unit vectors such that $\bar{a} \cdot \vec{b} = \bar{a} \cdot \bar{c} = 0$ and angle between $\vec{b}$ and $\bar{c}$ is $\frac{\pi}{6}$, prove that $\bar{a} = \pm 2 (\vec{b} \times \bar{c})$.

21. Show that the four points (0, -1, -1), (4, 5, 1), (3, 9, 4) and (-4, 4, 4) are coplanar. Also, find the equation of the plane containing them.

22. A coin is biased so that the head is 3 times as likely to occur as tail. If the coin is tossed three times, find the probability distribution of number of tails.

OR

How many times must a man toss a fair coin, so that the probability of having at least one head is more than 80%?

SECTION C

23. Using properties of determinants, show that

$$\Delta = \begin{vmatrix} (b+c)^2 & ab & ca \\ ab & (a+b)^2 & bc \\ ac & bc & (a+b)^2 \end{vmatrix} = 2abc\,(a+b+c)^3$$

24. The sum of the perimeter of a circle and a square is k, where k is some constant. Prove that the sum of their areas is least when the side of square is double the radius of the circle.

OR

A helicopter is flying along the curve $y = x^2+2$. A soldier is placed at the point (3, 2). Find the nearest distance between the soldier and the helicopter.

25. Evaluate : $\int \frac{1}{\sin x\,(5-4\cos x)}\,dx$

OR

Evaluate : $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\,dx$

26. Using integration, find the area of the region

$\left\{(x, y) : |x-1| \le y \le \sqrt{5-x^2}\right\}$

27. Show that the lines $\frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5}$ and $\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{5}$ are coplanar. Also find the equation of the plane.

28. From a pack of 52 cards, a card is lost. From the remaining 51 cards, two cards are drawn at random (without replacement) and are found to be both diamonds. What is the probability that the lost card was a card of heart?

29. A diet for a sick person must contain at least 4000 units of vitamins, 50 units of minerals and 1400 calories. Two foods X and Y are available at a cost of Rs 4 and Rs 3 per unit respectively. One unit of food X contains 200 units of vitamins, 1 unit of minerals and 40 calories, whereas 1 unit of food Y contains 100 units of vitamins, 2 units of minerals and 40 calories. Find what combination of foods X and Y should be used to have least cost, satisfying the requirements. Make it an LPP and solve it graphically.

MARKING SCHEME

MATHEMATICS CLASS - XII

SAMPLE PAPER II

SECTION A

1. 6
2. 4
3. 27
4. 17
5. 5x2
6. $\sin^{-1}\left(\frac{x}{2}\right)$
7. tanx
8. $-(3\hat{i}+2\hat{j}+7\hat{k})$
9. $\lambda = -9$
10. $\lambda = -3$ (1 mark for correct answer for Qs. 1 to 10)

SECTION B

11. Let x, y be any two elements of R (domain)

then $f(x) = f(y) \Rightarrow 2x^3-7 = 2y^3-7$

$\Rightarrow x^3=y^3 \Rightarrow x=y$ 1

so, f is an injective function

Let y be any element of R (co-domain)

$\therefore f(x) = y \Rightarrow 2x^3-7=y$

$$\Rightarrow x^3 = \frac{y+7}{2} \Rightarrow x = \left(\frac{y+7}{2}\right)^{\frac{1}{3}}$$

Now for all $y \in R$ (co-domain), there exists $x = \left(\frac{y+7}{2}\right)^{\frac{1}{3}} \in R$(domain 1

such that $f(x) = f\left\{\left(\frac{y+7}{2}\right)^{\frac{1}{3}}\right\} = 2\left\{\left(\frac{y+7}{2}\right)^{\frac{1}{3}}\right\}^3 - 7$

$$= 2\cdot\frac{y+7}{2}-7 = y$$ 1

so, f is surjective

Hence, f is a bijective function 1

OR

$\text{fog}\left(\frac{5}{2}\right) = f\left[g\left(\frac{5}{2}\right)\right] = f(2) = |2| = 2$ 2

$\text{gof}\left(-\sqrt{2}\right) = g\left[f\left(-\sqrt{2}\right)\right] = g\left[\left|-\sqrt{2}\right|\right] = g\left[\sqrt{2}\right] = 1$ 2

12. L.H.S. = $\tan^{-1}1 + \tan^{-2}2 + \tan^{-1}3$

$= \frac{\pi}{4} + \frac{\pi}{2} - \cot^{-1}2 + \frac{\pi}{2} - \cot^{-1}3$ ½

$= \frac{5\pi}{4} - \tan^{-1}\left(\frac{1}{2}\right) - \tan^{-1}\left(\frac{1}{3}\right)$ ½

$= \frac{5\pi}{4} - \left(\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3}\right)$ ½

$= \frac{5\pi}{4} - \tan^{-1}\left(\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\cdot\frac{1}{3}}\right)$ 1

$= \frac{5\pi}{4} - \tan^{-1}\ (1)$ ½

$= \frac{5\pi}{4} - \frac{\pi}{4}$ ½

$= \pi = \text{RHS}$ ½

13. $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix}$ 1

$A^2 - 5A - 14I = \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} - 5\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} - 14\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ½

$= \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} + \begin{bmatrix} -15 & 25 \\ 20 & -10 \end{bmatrix} + \begin{bmatrix} -14 & 0 \\ 0 & -14 \end{bmatrix}$

$$\begin{bmatrix} 29-15-14 & -25+25-0 \\ -20+20+0 & 24-10-14 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$ 1

Premultiplying $A^2-5A-14I = 0$ by A^{-1}, we get

$A^{-1}.A^2 - 5A^{-1}A - 14A^{-1}I = 0$

or, $A-5I - 14A^{-1} = 0$ ½

$$\text{or } A^{-1} = \frac{1}{14}(A-5I) = \frac{1}{14}\left\{\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} + \begin{bmatrix} -5 & 0 \\ 0 & -5 \end{bmatrix}\right\}$$

$$= \frac{1}{14}\begin{bmatrix} -2 & -5 \\ -4 & -3 \end{bmatrix}$$ 1

14. $f(x) = |(x-3)| \Rightarrow f(x) = \begin{cases} x-3 & \text{if } x \geq 3 \\ -(x-3) & \text{if } x < 3 \end{cases}$ ½

$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} -(x-3) = 0$ ½

$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (x-3) = 0$ ½

and $f(3) = 3-3 = 0$

$\therefore \lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x) = f(3)$

$\therefore$ f(x) is continuous at x = 3 ½

For differentiability

$$Lf'(3) = \lim_{x \to 3^-} \frac{f(x) - f(3)}{x-3} = \lim_{x \to 3^-} \frac{-(x-3) - 0}{x-3} = -1$$ ½

$$Rf'(3) = \lim_{x \to 3^+} \frac{f(x) - f(3)}{x-3} = \lim_{x \to 3^+} \frac{(x-3) - 0}{x-3} = 1$$ ½

$\therefore Lf'(3) \neq Rf'(3)$

so, f(x) is not differentiable at x=3 1

OR

$$\tan\left(\frac{x^2-y^2}{x^2+y^2}\right) = a$$

$$\Rightarrow \frac{x^2-y^2}{x^2+y^2} = \tan^{-1} a \qquad \text{---------(1)}$$ ½

Differentiating (1) w.r.t. x, we get

$$\frac{\left(x^2+y^2\right)\left(2x-2y\frac{dy}{dx}\right) - \left(x^2-y^2\right)\left(2x+2y\frac{dy}{dx}\right)}{\left(x^2+y^2\right)^2} = 0$$

or, $2x\left(x^2+y^2\right)-2y\left(x^2+y^2\right)\frac{dy}{dx} - 2x(x^2-y^2) - 2y(x^2-y^2)\frac{dy}{dx} = 0$ 2

or, $\frac{dy}{dx}\left[-2x^2y - \cancel{2y^3} - 2x^2y + \cancel{2y^3}\right] = \cancel{2x^3} - 2xy^2 + \cancel{2x^3} - 2xy^2$ 1

$$\Rightarrow \frac{dy}{dx}\left[-4x^2y\right] = -4xy^2$$

$\Rightarrow \frac{dy}{dx} = \frac{-4xy^2}{-4x^2y} = \frac{y}{x}$ ½

15. We know that e^x, sin x and cos x functions are continuous and differentiable everywhere. Product, sum and difference of two continuous functions is again a continuous function, so f is also continuous in $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$ 1

Now, $f\left(\frac{\pi}{4}\right) = e^{\frac{\pi}{4}}\left(\sin\frac{\pi}{4} - \cos\frac{\pi}{4}\right) = 0$

$f\left(\frac{5\pi}{4}\right) = e^{\frac{5\pi}{4}}\left(\sin\frac{5\pi}{4} - \cos\frac{5\pi}{4}\right) = 0$

$\Rightarrow f\left(\frac{\pi}{4}\right) = f\left(\frac{5\pi}{4}\right)$ 1

$\therefore$ Rolle's theorem is applicable

$f'(x) = e^x(\sin x - \cos x) + e^x(\cos x + \sin x) = 2e^x \sin x$

$\therefore$ $f'(x) = 0$ gives $2e^x \sin x = 0$

or $\sin x = 0 \Rightarrow x = 0, \pi$ 1

Now $\pi \in \left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$

$\therefore$ The theorem is verified with $x = \pi$ 1

16. Let $x = 25$, $x+\Delta x = 25\cdot 2$ so $\Delta x = 0\cdot 2$ 1

Let $y = \sqrt{x} \Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{25}} = \frac{1}{10}$ at $x = 25$ 1

$$dy = \frac{dy}{dx}\cdot \Delta x$$

or $\Delta y = \frac{dy}{dx}\cdot \Delta x = \frac{1}{10} \times 0.2 = 0.02$ 1

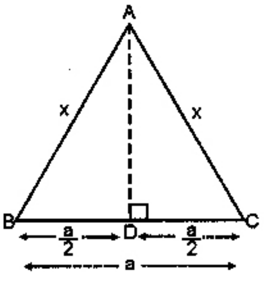
$\therefore \sqrt{25.2} = y+\Delta y = 5+0.02 = 5.02$ 1

OR

Let A be the area of Δ ABC in which AB = AC = x and BC = a

$\therefore A = \frac{1}{2} BC\times AD$

$= \frac{1}{2}a\sqrt{x^2-\frac{a^2}{4}} = \frac{a}{4}\sqrt{4x^2-a^2}$ 1



$$\frac{dA}{dt} = \frac{a}{4}\cdot\frac{1}{2\sqrt{4x^2-a^2}}\cdot 8x\cdot\frac{dx}{dt}$$ 1

$$= \frac{ax\times 9}{\sqrt{4x^2-a^2}}$$ 1

$\therefore \left(\frac{dA}{dt}\right)_{\text{at } x=a} = \frac{9a\cdot a}{\sqrt{3a^2}} = 3\sqrt{3}\ \text{cm}^2/\text{second}$ 1

17. $I = \int_{-1}^{1/2} |x\cos(\pi x)|dx$

Three cases arise :

Case I : $-1 < x < \frac{-1}{2}$

$\Rightarrow -\pi < \pi x < -\frac{\pi}{2}$

$\Rightarrow \cos\pi x < 0 \Rightarrow x\cos\pi x > 0$ ½

Case II : $-\frac{1}{2} < x < 0$

$-\frac{\pi}{2} < \pi x < 0$

$\Rightarrow \cos(\pi x) > 0$

$\Rightarrow x\cos(\pi x) < 0$ ½

case III : $0 < x < \frac{1}{2}$

$\Rightarrow 0 < \pi x < \frac{\pi}{2}$

$\Rightarrow \cos \pi x > 0$

$\Rightarrow x\cos \pi x > 0$ ½

$\therefore\ I = \int_{-1}^{-1/2} x\cos\pi x\, dx + \int_{-1/2}^{0} -x\cos\pi x dx + \int_{0}^{1/2} x\cos\pi x\, dx$ 1

$= \left[\frac{x\sin\pi x}{\pi} + \frac{\cos\pi x}{\pi^2}\right]_{-1}^{-\frac{1}{2}} - \left[\frac{x\sin\pi x}{\pi} + \frac{\cos\pi x}{\pi^2}\right]_{-\frac{1}{2}}^{0} + \left[\frac{x\sin\pi x}{\pi} + \frac{\cos\pi x}{\pi^2}\right]_{0}^{\frac{1}{2}}$ 1

$= \left[\left(\frac{1}{2\pi} + 0\right) - \left(0 - \frac{1}{\pi^2}\right)\right] - \left[-\left(\frac{1}{2\pi} + 0\right) + \left(0 + \frac{1}{\pi^2}\right)\right] + \left[-\left(0 + \frac{1}{\pi^2}\right) + \left(\frac{1}{2\pi} + 0\right)\right]$

$\frac{1}{2\pi} + \cancel{\frac{1}{\pi^2}} + \frac{1}{2\pi} - \cancel{\frac{1}{\pi^2}} + \frac{1}{2\pi} - \frac{1}{\pi^2}$

$= \frac{3}{2\pi} - \frac{1}{\pi^2}$ ½

18. $y\, e^{\frac{x}{y}}\, dx = \left(x\, e^{\frac{x}{y}} + y\right) dy$

$\Rightarrow \frac{dx}{dy} = \frac{x\, e^{\frac{x}{y}} + y}{y \cdot e^{\frac{x}{y}}}$ ½

Let $x = vy \Rightarrow \frac{dx}{dy} = v + y \cdot \frac{dv}{dy}$ ½

$$\therefore\ v + y\frac{dv}{dy} = \frac{vy\cdot e^v + y}{y\cdot e^v}$$ 1

$$\Rightarrow y\frac{dv}{dy} = \frac{vye^v + y}{y\cdot e^v} - v = \frac{\cancel{vye^v} + y - \cancel{vye^v}}{y\cdot e^v} = \frac{1}{e^v}$$ ½

$$\Rightarrow\ e^v dv = \frac{dy}{y}$$ 1

Integrating we get $e^v = \log y + \log c = \log cy$ 1

Substituting $v = \frac{x}{y}$, we get ½

$$e^{\frac{x}{y}} = \log cy$$

19. $(1+y+x^2y)dx + (x+x^3)dy = 0$

$$\Rightarrow\ x(1+x^2)dy = -[1+y(1+x^2)]dx$$

$$\Rightarrow\ \frac{dy}{dx} = \frac{-1-y(1+x^2)}{x(1+x^2)} = \frac{-1}{x}\cdot y - \frac{1}{x(1+x^2)}$$ 1

$$\text{or } \frac{dy}{dx} + \frac{1}{x}\cdot y = -\frac{1}{x(1+x^2)}$$

$$\therefore\ \text{I.F.} = \int e^{\frac{1}{x}dx} = e^{\log x} = x$$ 1

$\therefore$ The solution is

$$y.x = -\int\frac{1}{x(1+x^2)}\cdot x\,dx = -\int\frac{dx}{1+x^2}$$

$$= \tan^{-1}x + c$$ 1

when x=1, y=0

$$\therefore\ 0 = -\tan^{-1}(1) + c \qquad \Rightarrow c = \pi/4$$ ½

$$\therefore\ xy = -\tan^{-1}x + \frac{\pi}{4}$$ ½

20. $\vec{a}, \vec{b} = 0$ and $\vec{a}\cdot\vec{c} = 0$ ½

$\Rightarrow\ \vec{a}\perp\vec{b}$ and $\vec{a}\perp\vec{c}$

$\therefore$ $\vec{a}$ is $\perp$ to the plane of $\vec{b}$ and $\vec{c}$

$\Rightarrow$ $\vec{a}$ is parallel to $\vec{b}\times\vec{c}$ 1

Let $\vec{a} = k\,(\vec{b}\times\vec{c})$, where k is a scalar ½

$\therefore\ |\vec{a}| = |k|\ |\vec{b}\times\vec{c}|$

$= |k|\ |\vec{b}|\ |\vec{c}|\ \sin\frac{\pi}{6}$ 1

$\therefore\ 1 = |k|\frac{1}{2} \Rightarrow |k| = 2$ ½

$\therefore\ k = \pm 2$ ½

$\therefore\ \vec{a} = \pm 2(\vec{b}\times\vec{c})$

21. Equation of plane passing through (0, -1, -1) is

a(x-0) + b(y+1) +c(z+1) = 0 ---(i) ½

(i) passes through (4, 5, 1) and (3, 9, 4)

$\Rightarrow$ 4a+6b+2c = 0 or 2a+3b+c=0 ---(ii) 1

and 3a + 10b + 5c = 0 ---(iii)

from (ii) and (iii), we get

$$\frac{a}{15-10} = \frac{-b}{10-3} = \frac{c}{20-9} \Rightarrow \frac{a}{5} = \frac{-b}{7} = \frac{c}{11} = k \text{ (say)}$$ 1

$\therefore$ a = 5k, b = -7k, c = 11k ---(iv)

Putting these values of a, b, c in (i), we get

5kx - 7k(y+1) + 11k(z+1) = 0

or 5x-7y+11z+4 = 0 ---(v) ½

Putting the point (-4, 4, 4) in (v), we get

-20-28+44+4=0 which is satisfied ½

$\therefore$ The given points are co-planar and equation

of plane is 5x-7y+11z+4 = 0 ½

22. According to the given question

$P(H) = \frac{3}{4},\ P(T) = \frac{1}{4}$ ½

Let X be the random variate, which can take values 0, 1, 2, 3

$P(X=0) = P(\text{No Tails}) = P(HHH) = \frac{3}{4}\times\frac{3}{4}\times\frac{3}{4} = \frac{27}{64}$ ½

$P(X=1) = P(1\text{ Tail}) = P(HHT) + P(HTH) + P(THH)$

$= \frac{3}{4}\times\frac{3}{4}\times\frac{1}{4} + \frac{3}{4}\times\frac{1}{4}\times\frac{3}{4} + \frac{1}{4}\times\frac{3}{4}\times\frac{3}{4} = \frac{27}{64}$ 1

$P(X=2) = P(2\text{ tails}) = P(HTT) + P(THT) + P(TTH)$

$\frac{3}{4}\times\frac{1}{4}\times\frac{1}{4} + \frac{1}{4}\times\frac{3}{4}\times\frac{1}{4} + \frac{1}{4}\times\frac{1}{4}\times\frac{3}{4} = \frac{9}{64}$ 1

$P(X=3) = P(3\text{ tails}) = P(TTT)$

$\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4} = \frac{1}{64}$ ½

Reqd. Probability Distribution is

X	0	1	2	3
P(X)	27/64	27/64	9/64	1/64

OR

For a fair coin, $p(H) = \frac{1}{2}$ and $p(T) = \frac{1}{2}$ where H and T denote Head and Tail respectively. ½

Let the coin be tossed n times

$\therefore$ Required probability $= 1 - p\text{ (all Tails)}$

$= 1 - \frac{1}{2^n}$ ---(i) 1½

It has to be >80%

Total probability = 1 1

$\therefore$ (i) has to be $> \frac{4}{5}$

$\therefore 1 - \frac{1}{2^n} > \frac{4}{5} \Rightarrow n = 3$

$\therefore$ The fair coin has to be tossed 3 times for the desired situation. 1

SECTION C

23. $$\Delta = \begin{vmatrix} (b+c)^2 & ab & ca \\ ab & (a+c)^2 & bc \\ ac & bc & (a+b)^2 \end{vmatrix}$$

Operating $R_1 \to a R_1,\ R_2 \to b R_2,\ R_3 \to c R_3$, to get

$$\Delta = \frac{1}{abc}\begin{vmatrix} a(b+c)^2 & a^2b & a^2c \\ ab^2 & b(a+c)^2 & b^2c \\ ac^2 & b^2c & c(a+b)^2 \end{vmatrix} = \frac{abc}{abc}\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (a+c)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix}$$ 1+½

Operating $c_2 \to c_2 - c_1,\ c_3 \to c_3 - c_1$, to get

$$\Delta = \begin{vmatrix} (b+c)^2 & a^2-(b+c)^2 & a^2-(b+c)^2 \\ b^2 & (a+c)^2-b^2 & 0 \\ c^2 & 0 & (a+b)^2-c^2 \end{vmatrix} = (a+b+c)^2\begin{vmatrix} (b+c)^2 & a-b-c & a-b-c \\ b^2 & a+b-c & 0 \\ c^2 & 0 & a+b-c \end{vmatrix}$$ 1+½

Operating $R_1 \to R_1 - (R_2 + R_3)$ to get

$$\Delta = (a+b+c)^2\begin{vmatrix} 2bc & -2c & -2b \\ b^2 & a+c-b & 0 \\ c^2 & 0 & a+b-c \end{vmatrix} \quad c_2 \to c_2 + \frac{1}{b}c_1,\ c_3 \to c_3 + \frac{1}{c}c_1$$

$$= (a+b+c)^2\begin{vmatrix} 2bc & 0 & 0 \\ b^2 & a+c & \frac{b^2}{c} \\ c^2 & \frac{c^2}{b} & a+b \end{vmatrix}$$ 1+1

$(a+b+c)^2\left[2bc(a^2+ac+ab+bc-bc)\right] = (a+b+c)^2(2bc)\,a(a+b+c)$

$= (a+b+c)^3 \cdot 2abc$ 1

24. Let the radius of circle be r and side of square be x

$\therefore\ 2\pi r + 4x = k$ ---(A) 1

Let A be the sum of the areas of circle and square

$\therefore\ A = \pi r^2 + x^2$

$= \pi\left[\frac{k-4x}{2\pi}\right]^2 + x^2$ [using (A)] 1

$= \pi\left[\frac{k^2+16x^2-8kx}{4\pi^2}\right] + x^2$

$= \frac{k^2+16x^2-8kx}{4\pi} + x^2$ ½

$\therefore \frac{dA}{dx} = \frac{1}{4\pi}[0+32x-8k]+2x$

$= \frac{1}{4\pi}[32x-8k+8\pi x]$ ½

For optimisation $\frac{dA}{dx} = 0 \Rightarrow (32+8\pi x) = 8k$

$\Rightarrow x = \frac{k}{4+\pi}$ --(i) 1

$\therefore \frac{d^2A}{dx^2} = \frac{1}{4\pi}[32+8\pi] > 0 \Rightarrow$ Minima ½

Putting the value of x in (A) to get

$2\pi r + 4\cdot\frac{k}{4+\pi} = k$

$\Rightarrow 2\pi r = k - \frac{4k}{4+\pi} = \frac{\pi k}{4+\pi}$ 1

$2r = \frac{k}{4+\pi}$ --(ii)

From (i) and (ii), $x = 2r$ ½

OR

Let P(x,y) be the position of the Helicopter and the position of soldier at A(3, 2)

$\therefore AP = \sqrt{(x-3)^2+(y-2)^2} = \sqrt{(x-3)^2+(x^2)^2}$ $\left[\begin{array}{l}\therefore y=x^2+2 \text{ is the} \\ = \text{n of curve}\end{array}\right]$ 2

Let $AP^2 = z = (x-3)^2+x^4$

$\Rightarrow \frac{dz}{dx} = 2(x-3)+4x^3$ 1

For optimisation $\frac{dz}{dx} = 0 \Rightarrow 2x^3 + x - 3 = 0$

or $(x-1)(2x^2+2x+3) = 0 \Rightarrow x=1$ $\left[\begin{array}{l}\text{other factor} \\ \text{gives no real values}\end{array}\right]$ 1

$\frac{d^2z}{dx^2} = 6x^2 + 1 > 0 \Rightarrow$ Minima

when $x=1$, $y=x^2+2=3$

$\therefore$ The required point is (1, 3) 1

And distance AP = $\sqrt{(1-3)^2+(3-2)^2} = \sqrt{5}$ 1

25. $\int \frac{1}{\sin x\,(5-4\cos x)}\,dx = \int \frac{\sin x}{\sin^2 x\,(5-4\cos x)}\,dx$ ½

$= \int \frac{\sin x}{(1-\cos^2 x)\,(5-4\cos x)}\,dx$

$= -\int \frac{dt}{(1-t^2)\,(5-4t)}$, where $\cos x = t$, $dt = -\sin x\,dx$

$= -\int \frac{dt}{(1-t)\,(1+t)(5-4t)}$ 1

Let $\frac{1}{(1-t)\,(1+t)(5-4t)} = \frac{A}{1-t} + \frac{B}{1+t} + \frac{C}{5-4t}$ ½

$\Rightarrow 1 = A(1+t)(5-4t) + B(1-t)(5-4t) + c(1-t^2)$ --(i)

Putting $t = 1$ in (i) to get $A = \frac{1}{2}$

Putting $t = -1$ in (i) to get $B = \frac{1}{18}$ 1½

Putting $t = \frac{5}{4}$ in (i) to get $C = -\frac{16}{9}$

$\therefore I = -\left[\frac{1}{2}\int\frac{dt}{1-t} + \frac{1}{18}\int\frac{dt}{1+t} - \frac{16}{9}\int\frac{dt}{5-4t}\right]$

$$-\left[-\frac{1}{2}\log\ |1\text{-}t|+\frac{1}{18}\log|1\text{+}t|-\frac{16}{9x4}\log|5-4t|\right]+c$$ 1½

$$= \frac{1}{2}\log\ |1\text{-}\cos x|-\frac{1}{18}\log|1+\cos x|-\frac{4}{9}\log|5-4\cos x|+c$$ 1

OR

$$I=\int\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}dx=\int\frac{\sqrt{1-\sqrt{x}}\cdot\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}\sqrt{1-\sqrt{x}}}dx=\int\frac{1\text{-}\sqrt{x}}{\sqrt{1\text{-}x}}dx$$ 1½

$$=\int\frac{dx}{\sqrt{1\text{-}x}}-\int\frac{\sqrt{x}}{\sqrt{1\text{-}x}}dx\ =I_1\text{-}I_2$$ ½

$$I_1=\int(1\text{-}x)^{-\frac{1}{2}}dx=\text{-}2(1\text{-}x)^{\frac{1}{2}}+c_1\ \text{or}\ \text{-}2\sqrt{1\text{-}x}+c_1$$ 1

$$I_2=\int\frac{\sqrt{x}}{\sqrt{1\text{-}x}}dx : \text{Let } x=\sin^2\theta,\ dx=2\sin\theta\cos\theta\ d\theta$$ ½

$$=\int\frac{\sin\theta\cdot 2\sin\theta\cos\theta\ d\theta}{\cos\theta}=2\int\sin^2\theta\ d\theta$$ 1

$$=\int(1\text{-}\cos 2\theta)d\theta=\theta\text{-}\frac{\sin 2\theta}{2}=\theta\text{-}\sin\theta\cos\theta+c_2$$

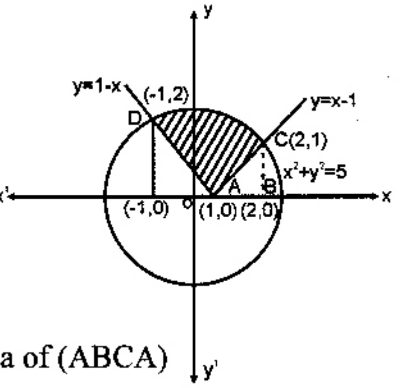
$$=\sin^{-1}\sqrt{x}-\sqrt{x}\ \sqrt{1\text{-}x}+c_2$$ 1

$$\therefore\ I=\text{-}2\sqrt{(1\text{-}x)}\ \text{-}\sin^{-1}\sqrt{x}+\sqrt{x}\ \sqrt{1\text{-}x}+C$$

$$=\sqrt{1\text{-}x}\ \left[\sqrt{x}-2\right]-\sin^{-1}\sqrt{x}+C$$ ½

26. Equations of curves are

$$x^2+y^2=5 \text{ and } y=\begin{cases}1\text{-}x, & x<1\\ x\text{-}1, & x>1\end{cases}$$ ½



correct figure --1

Points of intersection are c(2, 1) ½

D(-1, 2)

Required Area = Area of (EABCDE) - Area of (ADEA)- Area of (ABCA) ½

$$= \int_{-1}^{2} \sqrt{5-x^2}\,dx - \int_{-1}^{1} (1-x)\,dx - \int_{-1}^{2} (x-1)\,dx$$ 1

$$= \left[\frac{x}{2}\sqrt{5-x^2} + \frac{5}{2}\sin^{-1}\frac{x}{\sqrt{5}}\right]_{-1}^{2} - \left[x - \frac{x^2}{2}\right]_{-1}^{1} - \left[\frac{x^2}{2} - x\right]_{1}^{2}$$ 1

$$= \left[\left\{1 + \frac{5}{2}\sin^{-1}\frac{2}{\sqrt{5}}\right\} - \left\{-\frac{1}{2}\times 2 + \frac{5}{2}\sin^{-1}\left(\frac{-1}{\sqrt{5}}\right)\right\}\right] - \left[\left(1-\frac{1}{2}\right) - \left(-1-\frac{1}{2}\right)\right]$$
$$- \left[(2-2) - \left(\frac{1}{2} - 1\right)\right]$$ 1

$$= 1 + \frac{5}{2}\sin^{-1}\frac{2}{\sqrt{5}} + 1 - \frac{5}{2}\sin^{-1}\left(\frac{-1}{\sqrt{5}}\right) - 2 - \frac{1}{2}$$
$$= -\frac{1}{2} + \frac{5}{2}\left[\text{Sin}^{-1}\frac{2}{\sqrt{5}} - \sin^{-1}\left(\frac{-1}{\sqrt{5}}\right)\right]$$ ½

27. Lines $\frac{x-x_1}{l_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{n_1}$ and $\frac{x-x_2}{l_2} = \frac{y-y_2}{m_2} = \frac{z-z_2}{n_2}$ are 1½

coplanar if $\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$

In this case $\begin{vmatrix} -2 & -1 & 0 \\ -3 & 1 & 5 \\ -1 & 2 & 5 \end{vmatrix} = -2(5-10)+1(-15+5)+0 = 10-10 = 0$ 1½

∴ Lines are coplanar

Equation of plane containing this is

$\begin{vmatrix} x+3 & y-1 & z-5 \\ -3 & 1 & 5 \\ -1 & 2 & 5 \end{vmatrix} = 0$ 1½

$\Rightarrow 5x-10y+5z = 0$

or $x-2y+z = 0$ 1½

28. Let events E_1, E_2, E_3, E_4 and A be defined as follows

E_1 : Missing card is a diamond

E_2 : Missing card is a spade

E_3 : Missing card is a club 1

E_4 : Missing card is a heart

A : Drawing two diamond cards

$$P(E_1) = P(E_2) = P(E_3) = P(E_4) = \frac{1}{4}$$ ½

$$P\left(A/E_1\right) = \frac{12}{51} \times \frac{11}{50}$$ ½

$$P\left(A/E_2\right) = P\left(A/E_3\right) = P\left(A/E_4\right) = \frac{13}{51} \times \frac{12}{50}$$ 1

$$P\left(E_4/A\right) = \sum_{i=1}^{4} \frac{P(E_4) \cdot P\left(A/E_4\right)}{P(E_i) \cdot P\left(A/E_i\right)}$$ 1

$$\frac{\frac{1}{4} \cdot \frac{13}{51} \times \frac{12}{50}}{\frac{1}{4}\left[\frac{12\times11+13\times12+13\times12+13\times12}{51\cdot50}\right]}$$ 1

$$= \frac{13\times\cancel{12}}{3\times13\times\cancel{12}+\cancel{12}\times11} = \frac{13}{39+11} = \frac{13}{50}$$ 1

29. Let x and y be the units taken of Food A and Food B respectively then LPP is, ½

Minimise z = 4x+3y

Subject to constraints

$200x + 100y \geq 4000$ or $2x+y \geq 40$

$x+2y \geq 50$

$40x+40y \geq 1400$ or $x+y \geq 35$

$x \geq 0,\ y \geq 0$ 1½

Correct Graph 2

The corness of feasible region are

A(50,0), B(20,15), C (5, 30), D(0, 40) 1

$Z_A = 200$, $Z_B = 125$, $Z_C = 110$, $Z_D = 120$

$\therefore$ Z is minimum at C

$\therefore$ 5 units of Food A and 30 units of Food B

will give the minimum cost (which is Rs 110) 1