Chapter 11. Coordinate Geometry

Formulae

1. Distance Formula : The distance between the points P (x_1, y_1) and Q (x_2, y_2)

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

2. Section Formula : The co-ordinates of the point which divides the line segment joining the points P (x_1, y_1) and Q (x_2, y_2) in the ratio $m_1 : m_2$ are

$$\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

3. Mid-point Formula : The co-ordinates of the mid-point of the line segement joining the points P (x_1, y_1) and Q (x_2, y_2) are

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

4. Centroid Formula : The co-ordinates of the centroid of a triangle whose vertices are A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3) are

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

5. Slope of a Straight Line :

   (i) If the inclination of a line is θ ($\neq 90°$), its slope $= m = \tan\theta$.

   (ii) Slope of a line through (x_1, y_1) and (x_2, y_2) is given by $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.

6. Equation of a Straight Line :

   (i) Equation of a line parallel to x-axis is $y = b$.

   (ii) Equation of a line parallel to y-axis is $x = a$.

   (iii) Equation of a line with slope m and y-intercept c is $y = mx + c$.

   (iv) Equation of a line through (x_1, y_1) and with slope m is $y - y_1 = m(x - x_1)$.

7. Conditions of Parallelism and Perpendicularity : Two lines with slopes m_1 and m_2 are :

(i) parallel if and only if $m_1 = m_2$.

(ii) perpendicular if and only if $m_1 m_2 = -1$.

Formulae Based Questions

Question 1. Find the distance of the following points from origin.
(i) (5, 6) (ii) (a+b, a-b) (iii) (a cos θ, a sin θ).

Solution : (i) Let O(0, 0) be the origin.

Distance between O(0, 0) & P(5, 6)

$$OP = \sqrt{(5-0)^2 + (6-0)^2}$$
$$= \sqrt{25+36} = \sqrt{61} \text{ units.} \quad \text{Ans.}$$

(ii) Let P(a + b, a – b) and O(0, 0)

$$\text{Then } OP = \sqrt{(a+b-0)^2 + (a-b-0)^2}$$
$$= \sqrt{(a+b)^2 + (a-b)^2}$$
$$= \sqrt{a^2 + b^2 + 2ab + a^2 + b^2 - 2ab}$$
$$OP = \sqrt{2(a^2+b^2)} \text{ units.} \quad \text{Ans.}$$

(iii) Let P(a cos θ, a sin θ) and O(0, 0)

$$\text{Then } |OP| = \sqrt{(a\cos\theta - 0)^2 + (a\sin\theta - 0)^2}$$
$$= \sqrt{a^2\cos^2\theta + a^2\sin^2\theta}$$
$$= \sqrt{a^2(\sin^2\theta + \cos^2\theta)}$$
$$= a\sqrt{1} = a \text{ units.} \quad \text{Ans.}$$

Question 2. Calculate the distance between A (7, 3) and B on the x-axis, whose abscissa is 11.

Solution: Here B is (11, 0)

$$AB = \sqrt{(11-7)^2 + (0-3)^2}$$
$$= \sqrt{(4)^2 + (-3)^2}$$
$$= \sqrt{16+9}$$
$$= \sqrt{25} = 5 \text{ units.}$$

Question 3. KM is a straight line of 13 units If K has the coordinate (2, 5) and M has the coordinates (x, – 7) find the possible value of x.

Solution : Using distance formula

$$(x_2 - x_1)^2 + (y_2 - y_1)^2 = d^2$$

$$\Rightarrow \quad (x - 2)^2 + (-7 - 5)^2 = 13^2$$

$$\Rightarrow \quad x^2 - 4x + 4 + 144 = 169$$

$$\Rightarrow \quad x^2 - 4x + 148 - 169 = 0$$

$$\Rightarrow \quad x^2 - 4x - 21 = 0$$

$$\Rightarrow \quad x^2 - 7x + 3x - 21 = 0$$

$$\Rightarrow \quad (x - 7) + 3(x - 7) = 0$$

$$\Rightarrow \quad (x + 3)(x - 7) = 0$$

$$x = 7, -3.$$

Question 4. The midpoint of the line segment joining (2a, 4) and (-2, 2b) is (1, 2a+1). Find the value of a and b.

show that the points A(- 1, 2), B(2, 5) and C(- 5, – 2) are collinear.

Solution : Midpoint of (2a, 4) and (– 2, 2b) is (1, 2a + 1)

$x = \frac{x_1 + x_2}{2}$	$y = \frac{y_1 + y_2}{2}$
$1 = \frac{2a - 2}{2}$	$2a + 1 = \frac{4 + 2b}{2}$
$1 = a - 1$	$2a + 1 = 2 + b$
$\therefore \quad a = 2$	$\therefore \quad 5 - 2 = b$
	$\therefore \quad b = 3$

Therefore, a = 2, b = 3. Ans.

Question 5. Use distance formula to show that the points A(-1, 2), B(2, 5) and C(-5, -2) are collinear.

Solution : If using distance formula we have to prove that A, B and C are collinear, then we have to show :

$$BC = AC + AB$$

Hence $AB = \sqrt{(5-2)^2 + (2+1)^2} = \sqrt{9+9}$

$= \sqrt{18} = 3\sqrt{2}$ units.

$BC = \sqrt{(-2-5)^2 + (-5-2)^2}$

$= \sqrt{49 + 49} = \sqrt{98} = 7\sqrt{2}$ units.

and $AC = \sqrt{(-2-2)^2 + (-5+1)^2}$

$= \sqrt{16 + 16} = \sqrt{32} = 4\sqrt{2}$ units.

as $7\sqrt{2} = 4\sqrt{2} + 3\sqrt{2}$

$\Rightarrow \quad BC = AB + AC$

$\Rightarrow$ Points A,B and C are collinear.

Hence proved.

Determine the Following

Question 1. PQ is straight line of 13 units. If P has coordinate (2, 5) and Q has coordinate (x, – 7) find the possible values of x.

Solution : Here $PQ = 13$

$$PQ^2 = 13^2$$

$$\therefore \quad (x-2)^2 + (-7-5)^2 = 169$$

$$\Rightarrow \quad (x-2)^2 = 169 - 144$$

$$= 25 = 5^2$$

or $(x-2) = \pm 5$

$$\Rightarrow \quad x = 7 \text{ or } -3.$$

Question 2. Give the relation that must exist between x and y so that (x, y) is equidistant from (6, -1) and (2, 3).

Solution : Let $P(x, y)$. $A(6, -1)$ and $B(2, 3)$ be the given points.

Since $PA = PB$

So $\sqrt{(x-6)^2 + (y+1)^2} = \sqrt{(x-2)^2 + (y-3)^2}$

Squaring both sides

$$x^2 + 36 - 12x + y^2 + 1 + 2y = x^2 + 4 - 4x + y^2 + 9 - 6y$$

$$\Rightarrow \quad -8x + 8y = 13 - 36 - 1$$

$$\Rightarrow \quad -8(x-y) = -24$$

$$\Rightarrow \quad x - y = 3.$$

Ans.

Question 3. The line segment joining A (2, 3) and B (6, – 5) is intersected by the X axis at the point K. Write the ordinate of the point K. Hence find the ratio in which K divides AB.

Solution: A (2, 3) and B (6,- 5)

Intersected at X axis at K.

$\therefore$ y = 0 or ordinate = 0

K (x, 0)

Let required ratio be a : 1

$\therefore$ Ordinate of K = 0

$$0 = \frac{a \times -5 + 1 \times 3}{a+1}$$

$$0 = -5a + 3$$

$$5a = 3, \; a = \frac{3}{5}$$

$\therefore$ K divides AB in ratio of 3 : 5.

Question 4. Find the value of x so that the line passing through (3, 4) and (x, 5) makes an angle 135° with positive direction of X-axis.

Solution : Slope of the line which makes an angle 135° with X-axis,

$$m = \tan 135° = -1.$$

Also slope $m = \frac{5-4}{x-3} = \frac{1}{x-3}$

Then, $\frac{1}{x-3} = -1$

$\Rightarrow x - 3 = -1$

$\therefore x = 2.$ Ans.

Question 5. Find the value, of k, if the line represented by kx – 5y + 4 = 0 and 4x – 2y + 5 = 0 are perpendicular to each other.

Solution : Here, $kx - 5y + 4 = 0$

$\Rightarrow y = \frac{kx}{5} + \frac{4}{5}$

$\therefore$ The slope of the line is $\frac{k}{5}$.

Also $4x - 2y + 5 = 0$

$y = 2x + \frac{5}{2}$

$\therefore$ The slope of line is 2.

Since, the given lines are perpendicular to each other, we have

$\left(\frac{k}{5}\right)(2) = -1 \Rightarrow k = \frac{-5}{2}.$ Ans.

Question 6. Find the equation of a line which is inclined to x axis at an angle of 60° and its y – intercept = 2.

Solution : Hence, $m = \tan 60° = \sqrt{3}$

and $c = 2$

The equation of line is given by

$$y = mx + c$$

$$y = \sqrt{3}\,.\,x + 2$$

$$y = \sqrt{3}\,x + 2$$

$\sqrt{3}\,x - y + 2 = 0.$ Ans.

Question 7. Find the equation of a line with slope 1 and cutting off an intercept of 5 units on Y-axis.

Solution: We have

Slope of the line m = 1

and Y-intercept, $c = 5$ units

The equation of line is given by

$$y = mx + c$$

i.e., $y = 1.x + 5$

$\Rightarrow y = x + 5$

or $x - y + 5 = 0.$

Question 8. Find the equations of a line passing through the point (2, 3) and having the x – interecpt of 4 units.

Solution : Since x-interecpt is 4 units coordinates of point are (4, 0). Equation of a line passing through (2, 3) and (4, 0) is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\Rightarrow \quad y - 3 = \frac{0-3}{4-2}(x - 2)$$

$$\Rightarrow \quad y - 3 = \frac{-3}{2}(x - 2)$$

$$\Rightarrow \quad 2y - 6 = -3x + 6$$

$$\Rightarrow \quad 3x + 2y = 12.$$

Ans.

Question 9. The line through A (- 2, 3) and B (4, b) is perpendicular to the line 2a – 4y = 5. Find the value of b.

Solution : Slope of AB $= \frac{b-3}{4+2}$

$$m_1 = \frac{b-3}{6}$$

$$2x - 4y = 5$$

$$\Rightarrow \quad 4y = 2x - 5$$

$$\Rightarrow \quad y = \frac{1}{2}x - \frac{5}{4}$$

$$\text{Slope } (m_2) = \frac{1}{2}$$

Since both lines are perpendicular to each other

so,
$$m_1 \cdot m_2 = -1$$

$$\frac{b-3}{6} \cdot \frac{1}{2} = -1$$

$$b - 3 = -12$$

$$b = -9.$$

Ans.

Question 10. Given that $(a, 2a)$ lies on line $\frac{y}{2} = 3x - 6$. Find the value of a.

Solution : Point $(a, 2a)$ lies on the line

$$\frac{y}{2} = 3x - 6$$

$$\therefore \quad \frac{2a}{2} = 3a - 6$$

$$\Rightarrow \quad a = 3a - 6$$

$$\Rightarrow \quad 2a = 6$$

$$\Rightarrow \quad a = 3.$$

Ans.

Question 11. Find the equation of a straight line which cuts an intercept of 5 units on Y-axis and is parallel to the line joining the points (3, – 2) and (1, 4).

Solution : Let m be the slope of the required line and since the required line is parallel to the line joining the points (3, – 2) and (1, 4).

Hence, slope of the line

$$m = \frac{4+2}{1-3}$$

$$= \frac{6}{-2}$$

$$= -3.$$

Also, Y-intercept $c = 5$ units.

So, equation of the required line be

$$y = mx + c$$

$$\Rightarrow \quad y = -3x + 5$$

$$\Rightarrow \quad 3x + y - 5 = 0.$$

Ans.

Question 12. Find the equation of a line that has Y-intercept 3 units and is perpendicular to the line joining (2, – 3) and (4, 2).

Solution : Let m be the slope of required line.

Slope of the given line $= \frac{2+3}{4-2} = \frac{5}{2}$.

But the required line is perpendicular to the given line.

Hence,

$m \times$ Slope of the given line $= -1$

$$\Rightarrow \quad m \times \frac{5}{2} = -1$$

$$\Rightarrow \quad m = \frac{-2}{5}$$

Y-intercept, $c = 3$

Hence, equation of the required line is given by

$$y = mx + c$$

i.e.,

$$y = -\frac{2}{5}x + 3$$

$$\Rightarrow \quad 5y = -2x + 15$$

$$\Rightarrow \quad 2x + 5y - 15 = 0.$$

Ans.

Question 13. Find a general equation of a line which passes through:
(i) (0, -5) and (3, 0) (ii) (2, 3) and (-1, 2).

Solution : We have the equation of a line which passes through (x_1, y_1) and (x_2, y_2) is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

(i) Putting $x_1 = 0, y_1 = -5$ and $x_2 = 3, y_2 = 0$

$$y - (-5) = \frac{0-(-5)}{3-0}(x-0)$$

$$\Rightarrow \quad y + 5 = \frac{5}{3}(x-0)$$

$$\Rightarrow \quad 3y + 15 = 5x$$

$$\Rightarrow \quad 5x - 3y - 15 = 0$$

which is the required equation. Ans.

(ii) Putting $x_1 = 2, y_1 = 3$ and $x_2 = -1, y_2 = 2$

$$y - 3 = \frac{2-3}{-1-2}(x-2)$$

$$\Rightarrow \quad y - 3 = \frac{-1}{-3}(x-2)$$

$$\Rightarrow \quad 3y - 9 = (x-2)$$

$$\Rightarrow \quad x - 2 - 3y + 9 = 0$$

$$\Rightarrow \quad x - 3y + 7 = 0$$

which is the equation of the required line.

$$\Rightarrow \quad 3y - 9 = (x-2)$$

$$\Rightarrow \quad x - 2 - 3y + 9 = 0$$

$$\Rightarrow \quad x - 3y + 7 = 0$$

which is the equation of the required line.

Question 14. Find the equation of the line passing through (0, 4) and parallel to the line 3x + 5y + 15 = 0.

Solution : Since line is parallel to

$$3x + 5y + 15 = 0$$
$$5y = -3x - 15$$
$$y = \frac{-3}{5}x - 3$$
$$\therefore \quad m_1 = \frac{-3}{5}$$
$$m_1 = m_2 \quad (\because \text{ lines are parallel})$$
$$\therefore \quad m_2 = \frac{-3}{5}$$

and passing point = (0, 4)

Equation of line

$$y - y_1 = m(x - x_1)$$
$$\Rightarrow \quad y - 4 = \frac{-3}{5}(x - 0)$$
$$\Rightarrow \quad 5y - 20 = -3x$$
$$\Rightarrow \quad 3x + 5y = 20. \qquad \text{Ans.}$$

Question 15. Find the equation of a line passing through (3, – 2) and perpendicular to the line.

$$x - 3y + 5 = 0.$$

Solution : $x - 3y + 5 = 0$

$$\Rightarrow \quad 3y = x + 5$$
$$\therefore \quad y = \frac{x}{3} + \frac{5}{3}$$
$$\therefore \quad m_1 = \frac{1}{3}$$

Since lines are perpendicular to each other

$$\therefore \quad m_1 \times m_2 = -1$$
$$\frac{1}{3} \times m_2 = -1$$
$$m_2 = -1 \times 3$$
$$m_2 = -3$$

Passing point is (3, – 2)

∴ Equation of line

$$y - y_1 = m(x - x_1)$$
$$\Rightarrow \quad y + 2 = -3(x - 3)$$
$$\Rightarrow \quad y + 2 = -3x + 9$$
$$\Rightarrow \quad 3x + y + 2 - 9 = 0$$
$$\Rightarrow \quad 3x + y = 7.$$

Question 16. Find the equation of the straight line which has Y-intercept equal to 4/3 and is perpendicular to 3x – 4y + 11 = 0.

Solution : Equation of the given line is

$$3x - 4y + 11 = 0$$

Slope of this line $y = mx + c$

$$4y = 3x + 11$$

$$y = \frac{3}{4}x + \frac{11}{4}$$

$$m_1 = \frac{3}{4}$$

Let m_2 be the slope of the line which is perpendicular to the given line then

$$m_1 m_2 = -1$$

$$\frac{3}{4} m_2 = -1$$

$$\Rightarrow \quad m_2 = -\frac{4}{3}.$$

Also, Y-intercept $c = \frac{4}{3}$.

Equation of the required line

$$y = m_2 x + c$$

$$y = \frac{-4}{3}x + \frac{4}{3}$$

$$\Rightarrow \quad 3y = -4x + 4$$

$$\Rightarrow \quad 4x + 3y - 4 = 0. \qquad \text{Ans.}$$

Question 17. Find the equation of the straight line perpendicular to 5x – 2y = 8 and which passes through the mid-point of the line segment joining (2, 3) and (4, 5).

Solution : $5x - 2y = 8$

$$2y = 5x - 8$$

$$\Rightarrow \quad y = \frac{5}{2}x - 4$$

$$y = mx + c$$

$$\therefore \quad m_1 = \frac{5}{2}$$

Since lines are perpendicular to each other

$$\therefore \quad m_1 \times m_2 = -1$$

$$\frac{5}{2} \times m_2 = -1$$

$$m_2 = -1 \div \frac{2}{5}$$

$$m_2 = -\frac{2}{5}$$

Coordinates of midpoints

$$= \frac{2+4}{2}, \frac{3+5}{2}$$

Passing Point = (3, 4)

∴ Equation of line,

$$y - y_1 = m(x - x_1)$$

$$\Rightarrow \quad y - 4 = \frac{-2}{5}(x - 3)$$

$$\Rightarrow \quad 5y - 20 = -2x + 6$$

$$\Rightarrow \quad 2x + 5y = 26.$$

Question 18. A line passing through the points (a, 2a) and (- 2, 3) is perpendicular to the line 4a + 3y + 5 = 0. Find the value of a.

Solution : Let m_1 be the slope of the line joining at the points $(a, 2a)$ and $(-2, 3)$, then

$$m_1 = \frac{2a-3}{a+2}$$

Also slope of the line $4x + 3y + 5 = 0$

$$m_2 = -\frac{4}{3}$$

Since, both the lines are perpendicular.

So, $m_1 m_2 = -1$

$\Rightarrow \quad \frac{2a-3}{a+2} \times \frac{(-4)}{3} = -1$

$\Rightarrow \quad 8a - 12 = 3a + 6$

$\Rightarrow \quad 8a - 3a = 18$

$\Rightarrow \quad 5a = 18$

$\Rightarrow \quad a = \frac{18}{5}$

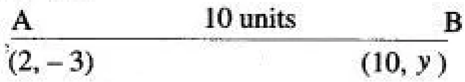
$\Rightarrow \quad a = 3\frac{3}{5}$.

Prove the Following

Question 1. A line is of length 10 units and one end is at the point (2, – 3). If the abscissa of the other end be 10, prove that its ordinate must be 3 or – 9.

Solution : Let AB be the line of length 10 units

then A (10, y)



$\Rightarrow \quad \sqrt{(2-10)^2 + (-3-y)^2} = 100$

Squaring both sides

$\Rightarrow \quad 64 + 9 + y^2 + 6y = 100$

$\Rightarrow \quad y^2 + 6y + 73 - 100 = 0$

$\Rightarrow \quad y^2 + 6y - 27 = 0$

$\Rightarrow \quad y^2 + 9y - 3y - 27 = 0$

$\Rightarrow \quad y(y+9) - 3(y+9) = 0$

$\Rightarrow \quad (y+9)(y-3) = 0$

$\Rightarrow \quad y = -9$ or $y = 3$

Ordinate is 3 or – 9. Hence proved.

Question 2. Show that the line joining (2, – 3) and (- 5, 1) is:

(i) Parallel to line joining (7, -1) and (0, 3).

(ii) Perpendicular to the line joining (4, 5) and (0, -2).

Solution : Let m_1 be the slope of line joining (2, – 3) and (– 5, 1) then

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{1-(-3)}{-5-2} = -\frac{4}{7}$$

(i) Let m_2 be the slope of the line joining (7, – 1) and (0, 3), then

$$m_2 = \frac{3-(-1)}{0-7} = -\frac{4}{7}$$

Since, $m_1 = m_2$, the two lines are parallel.

Hence proved.

(ii) Let m_3 be the slope of the line joining (4, 5) and (0, – 2) then

$$m_3 = \frac{-2-5}{0-4} = \frac{7}{4}$$

Now $$m_1 m_3 = -\frac{4}{7} \times \frac{7}{4} = -1$$

Hence, the two lines are perpendicular.

Hence proved.

Question 3. With out Pythagoras theorem, show that A(4, 4), B(3, 5) and C(-1, -1) are the vertices of a right angled.

Solution : Slope of BC = $m_1 = \frac{-1-5}{-1-3} = \frac{3}{2}$

Slope of CA = $m_2 = \frac{4-(-1)}{4-(-1)} = 1$

Also, slope of AB = $m_3 = \frac{5-4}{3-4} = -1$

Since, $m_2 m_3 = 1 \times (-1) = -1$, So, AB and CA are perpendicular to each other.

Thus, ΔABC is a right angled trianlge at A.

Hence proved.

Question 4. Show that the points A(- 2, 5), B(2, – 3) and C(0, 1) are collinear.

Solution : m_1 = Slope of AB

$$= \frac{-3-5}{2-(-2)} = -\frac{8}{4} = -2$$

m_2 = slop of BC

$$= \frac{1-(-3)}{0-2} = \frac{4}{-2} = -2.$$

Hence $m_1 = m_2 = -2$

So, AB is parallel to BC.

But B is common to AB and BC.
Hence, A, B and C must lies on the same line.

Hence proved.

Question 5. By using the distance formula prove that each of the following sets of points are the vertices of a right angled triangle.
(i) (6, 2), (3, -1) and (- 2, 4)
(ii) (-2, 2), (8, -2) and (-4, -3).

Solution : (i) Let A(6, 2), B(3, – 1) and C(– 2, 4) be the given points

$$AB = \sqrt{(6-3)^2 + (2+1)^2}$$
$$= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2} \text{ units.}$$
$$BC = \sqrt{(3+2)^2 + (-1-4)^2}$$
$$= \sqrt{25+25} = 5\sqrt{2} \text{ units.}$$
$$AC = \sqrt{(6+2)^2 + (2-4)^2}$$
$$= \sqrt{64+4}$$
$$= \sqrt{68} \text{ units.}$$

Now $AB^2 + BC^2 = (3\sqrt{2})^2 + (5\sqrt{2})^2$
$= 18 + 50 = 68 = AC^2$
$AB^2 + BC^2 = AC^2$.

Hence, the triangle is right angled at A.

Hence proved

(ii) Let A (– 2, 2), B(8, – 2) and C(– 4, – 3) be the given points

$$AB = \sqrt{(8+2)^2 + (-2-2)^2}$$
$$= \sqrt{116} \text{ units.}$$
$$BC = \sqrt{(8+4)^2 + (-2+3)^2}$$
$$= \sqrt{145} \text{ units.}$$
$$AC = \sqrt{(-2+4)^2 + (2+3)^2}$$
$$= \sqrt{29} \text{ units.}$$

Now $AB^2 + AC^2 = 116 + 29$
$= 145 = BC^2$

Since $AB^2 + AC^2 = BC^2$

Hence, the triangle is right angled at *A*.

Question 6. Show that each of the triangles whose vertices are given below are isosceles :
(i) (8, 2), (5,-3) and (0,0)
(ii) (0,6), (-5, 3) and (3,1).

Solution : (i) Let A (8, 2), B (5, – 3) and C (0, 0) be the given point

Then $AB = \sqrt{(8-5)^2 + (2+3)^2}$

$= \sqrt{9+25} = \sqrt{34}$ units.

$BC = \sqrt{(5-0)^2 + (-3-0)^2}$

$= \sqrt{25+9} = \sqrt{34}$ units.

$AC = \sqrt{(8-0)^2 + (2-0)^2}$

$= \sqrt{64+4} = \sqrt{68}$. units.

Here $AB = BC = \sqrt{34}$.

Hence, the triangle is isosceles.

(ii) The given points are A(0, 6), B(– 5, 3) and C(3, 1)

Then $AB = \sqrt{(0+5)^2 + (6-3)^2}$

$= \sqrt{25+9} = \sqrt{34}$ units.

$BC = \sqrt{(-5-3)^2 + (3-1)^2}$

$= \sqrt{64+4} = \sqrt{68}$ units.

Also $AC = \sqrt{(0-3)^2 + (6-1)^2}$

$= \sqrt{9+25} = \sqrt{34}$ units.

Since $AB = AC = \sqrt{34}$

Hence, the triangle is an isosceles.

Hence proved.

Question 7. Show that the quadrilateral with vertices (3, 2), (0, 5), (- 3, 2) and (0, -1) is a square.

Solution : Let A(3, 2), B(0, 5), C(– 3, 2) and D(0, – 1) are the vertices of quadrilateral.

Now $AB = \sqrt{(3-0)^2 + (2-5)^2}$

$= \sqrt{18}$ units.

$\Rightarrow AB^2 = 18$

$BC = \sqrt{(0+3)^2 + (5-2)^2}$

$= \sqrt{18}$ units.

$\Rightarrow BC^2 = 18$

$CD = \sqrt{(-3-0)^2 + (2+1)^2}$

$= \sqrt{18}$ units.

$\Rightarrow CD^2 = 18$

Also $AD = \sqrt{(3-0)^2 + (2+1)^2}$

$= \sqrt{18}$ units.

$\Rightarrow AD^2 = 18.$

Here $AB = BC = CD = DA$

$= \sqrt{18}$ units.

Also $AC^2 = (3+3)^2 + (2-2)^2$

$AC^2 = 36$

or $BD^2 = (0-0)^2 + (5+1)^2$

$= 36$

⇒ Diagonal AC = BD

Hence, ABCD is a square. Hence proved.

Question 8. Show that the points (a, a), $(-a, -a)$ and $(-a\sqrt{3}, a\sqrt{3})$ are the vertices of an equilateral triangle.

Solution : The given points are let

$A(a, a)$, $B(-a, -a)$ and $C(-a\sqrt{3}, a\sqrt{3})$.

$$AB = \sqrt{(-a-a)^2 + (-a-a)^2}$$
$$= \sqrt{4a^2 + 4a^2} = 2\sqrt{2}\, a \text{ units.}$$
$$BC = \sqrt{(-a\sqrt{3}+a)^2 + (a\sqrt{3}+a)^2}$$
$$= \sqrt{3a^2 + a^2 - 2\sqrt{3}a^2 + 3a^2 + a^2 + 2\sqrt{3}a^2}$$
$$= \sqrt{8a^2} = 2\sqrt{2}a \text{ units.}$$
and $$CA = \sqrt{(a\sqrt{3}-a)^2 + (-a\sqrt{3}-a)^2}$$
$$= \sqrt{3a^2 + a^2 - 2\sqrt{3}a^2 + 3a^2 + a^2 + 2\sqrt{3}a^2}$$
$$= \sqrt{8a^2} = 2\sqrt{2}a \text{ units.}$$
as $AB = BC = CA = 2\sqrt{2}\,a$

⇒ ΔABC is an equilateral triangle.

Hence proved.

Question 9. If the point (x, y) is at equidistant from the point (a + b, b – a) and (a-b, a + b). Prove that ay = bx.

Solution : Given that (x, y) is equidistant from the points $(a+b, b-a)$ and $(a-b, a+b)$.

Hence, distance of (x, y) from both points will be same.

Hence, $$\sqrt{(y-b+a)^2 + (x-a-b)^2} = \sqrt{(y-a-b)^2 + (x-a+b)^2}$$

On squaring and expanding :

$$y^2 + b^2 + a^2 - 2by - 2ab + 2ay + x^2 + a^2 + b^2 - 2ax + 2ab - 2bx$$
$$= y^2 + a^2 + b^2 - 2ay + 2ab - 2by + x^2 + a^2 + b^2 - 2ax - 2ab + 2bx$$
$$2ay - 2bx = 2bx - 2ay$$
$$4ay = 4bx$$
$$\Rightarrow \boxed{ay = bx}$$ Hence proved.

Question 10. Prove that A(4, 3), B(6, 4), C(5, 6) and D(3, 5) are the angular points of a square.

Solution : Now

$$AB = \sqrt{(4-6)^2 + (3-4)^2}$$

$$= \sqrt{4+1} = \sqrt{5} \text{ units}$$

$$BC = \sqrt{(6-5)^2 + (4-6)^2} = \sqrt{1+4}$$

$$BC = \sqrt{5} \text{ units}$$

$$CD = \sqrt{(5-3)^2 + (6-5)^2} = \sqrt{4+1}$$

$$CD = \sqrt{5} \text{ units}$$

Also $$DA = \sqrt{(4-3)^2 + (3-5)^2}$$

$$= \sqrt{1+4} = \sqrt{5}$$

$$DA = \sqrt{5} \text{ units}$$

So $$AB = BC = CD = DA.$$

Now slope of AB $= m_1 = \dfrac{4-3}{6-4} = \dfrac{1}{2}$

Slope of BC $= m_2 = \dfrac{6-4}{5-6} = \dfrac{2}{-1}$

Slope of CD $= m_3 = \dfrac{5-6}{3-5} = \dfrac{1}{2}$

Slope of DA $= m_4 = \dfrac{5-3}{3-4} = \dfrac{2}{-1}$

Since $m_1 = m_3$ and $m_2 = m_4$

So $AB \parallel CD$

and $BC \parallel DA$.

Also, $m_1 m_2 = \dfrac{1}{2} \times \dfrac{2}{-1} = -1$

Therefore, $AB \perp BC$

$\therefore$ ABCD is a square. Hence proved.

Question 13. P and Q are two points whose coordinate are $(at^2, 2at)$, $\left(\frac{a}{t^2}, \frac{-2a}{t}\right)$ and S is the point (a, 0). Prove that $\frac{1}{SP}+\frac{1}{SQ}$ is constant for all values of t.

Solution : The given points are $P(at^2, 2at)$, $Q\left(\frac{a}{t^2}, \frac{-2a}{t}\right)$ and $S(a, 0)$

Now,
$$SP = \sqrt{(at^2-a)^2+(2at-0)^2}$$
$$= \sqrt{a^2(t^4+1-2t^2)+4a^2t^2}$$
$$= a\sqrt{t^4+1+2t^2}$$
$$= a\sqrt{(t^2+1)^2}$$
$$SP = a(t^2+1) \text{ units.}$$

Also,
$$SQ = \sqrt{\left(\frac{a}{t^2}-a\right)^2+\left(\frac{-2a}{t}-0\right)^2}$$
$$= \sqrt{a^2\left(\frac{1}{t^4}+1-\frac{2}{t^2}\right)+\frac{4a^2}{t^2}}$$
$$= a\sqrt{\frac{1}{t^4}+1+\frac{2}{t^2}}$$
$$= a\sqrt{\left(\frac{1}{t^2}+1\right)^2}$$

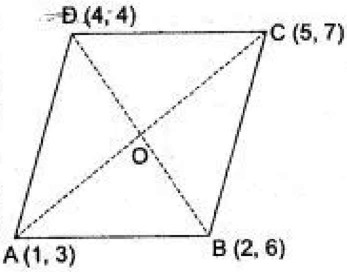
Question 14. Show that the points A(1, 3), B(2, 6), C(5, 7) and D(4, 4) are the vertices of a rhombus.

$$= a\left(\frac{1}{t^2}+1\right) = \frac{a(t^2+1)}{t^2}$$

Now
$$\frac{1}{SP}+\frac{1}{SQ} = \frac{1}{a(t^2+1)}+\frac{1\times t^2}{a\,(t^2+1)}$$
$$= \frac{(1+t^2)}{a(t^2+1)}$$
$$\frac{1}{SP}+\frac{1}{SQ} = \frac{1}{a}.$$

Solution : To show that ABCD is a rhombus, it is sufficient to show that

(i) ABCD is a parallelogram *i.e.,* AC and BD have the same mid point.



(ii) A pair of adjacent sides are equal.

Now, midpoint of AC $= \left(\frac{1+5}{2}, \frac{3+7}{2}\right)$

$= (3, 5).$

Midpoint of BD $= \left(\frac{4+2}{2}, \frac{4+6}{2}\right)$

$= (3, 5).$

Thus, ABCD is a parallelogram.

Also $AB^2 = (2-1)^2 + (6-3)^2$

$= 1 + 9 = 10$ units.

$BC^2 = (5-2)^2 + (7-6)^2$

$= 9 + 1 = 10$ units.

Therefore $AB^2 = BC^2 \Rightarrow AB = BC$

Hence, ABCD is a rhombus.

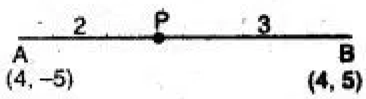
Figure Based Questions

Question 1. If the line joining the points A (4, – 5) and B(4, 5) is divided by the point P such that $\frac{AP}{AB} = \frac{2}{5}$, find the coordinates of P.

Solution : A(4, – 5), B(4, 5)

Given $\frac{AP}{AB} = \frac{2}{5}$

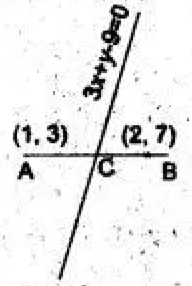
$\therefore \quad \frac{AP}{PB} = \frac{2}{3}$



Let coordinate of P(*x*, *y*).

Question 2. Determine the ratio in which the line 3x + y – 9 = 0 divides the line joining (1, 3) and (2, 7).

Solution : Suppose the line $3x + y - 9 = 0$ divides the line joining A(1, 3) and B(2, 7) in the ratio of $\lambda : 1$ at point C.



Coordinates of $C = \left(\frac{2\lambda+1}{\lambda+1}, \frac{7\lambda+3}{\lambda+1}\right)$

But point C lies on the line $3x + y - 9 = 0$.

Therefore,

$$3\left(\frac{2\lambda+1}{\lambda+1}\right)+\left(\frac{7\lambda+3}{\lambda+1}\right)-9 = 0$$

$$\Rightarrow \quad 6\lambda+3+7\lambda+3-9\lambda-9 = 0$$

$$\Rightarrow \quad 4\lambda-3 = 0$$

$$\Rightarrow \quad \lambda = \frac{3}{4}$$

The required ratio $= \lambda : 1$

$= 3 : 4.$ **Ans.**

Question 3. The midpoint of the line segment AB shown in the diagram is (4, – 3). Write down the coordinates of A and B.

Solution: Let the coordinates of A and Bare (x, 0) and (0, y).

$$\text{so} \quad x = \frac{mx_2+nx_1}{m+n}, \quad y = \frac{my_2+ny_1}{m+n}$$

$$m = 2, n = 3$$

$$x_1 = 4, x_2 = 4$$

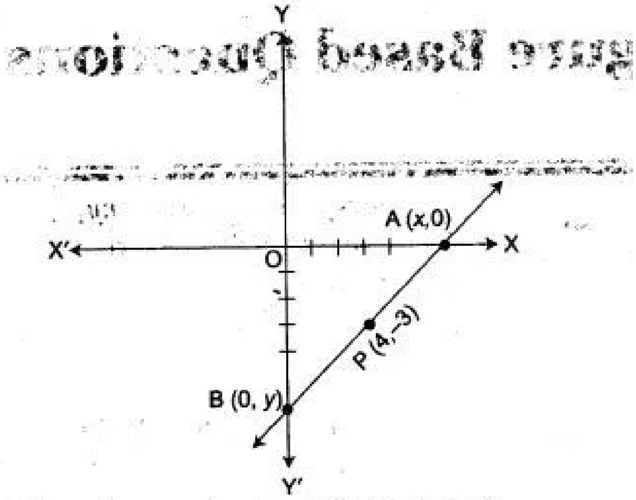
$$y_1 = -5, y_2 = 5$$

$$\therefore \quad x = \frac{2\times4+3\times4}{2+3} = \frac{8+12}{5} = \frac{20}{5} = 4$$

$$y = \frac{2\times5+3\times-5}{2+3}$$

$$= \frac{10-15}{5} = \frac{-5}{5} = -1$$

$\therefore$ Co-ordinates of P are $(4, -1)$. **Ans.**



Thus, the coordinates of midpoints of

$$AB = \left(\frac{x+0}{2}, \frac{y+0}{2}\right)$$

$$= \left(\frac{x}{2}, \frac{y}{2}\right)$$

According to question, the coordinates of mid-point = (4, – 3)

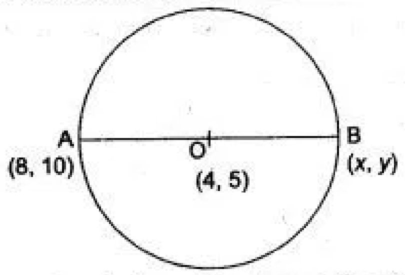
$$\therefore \quad \frac{x}{2} = 4, \quad x = 8$$

$$\frac{y}{2} = -3, \; y = -6$$

∴ The required points are (8, 0) and (0, – 6).

Question 4. The centre 'O' of a circle has the coordinates (4, 5) and one point on the circumference is (8, 10). Find the coordinates of the other end of the diameter of the circle through this point.

Solution : Let (x, y) be the coordinates of the other end of the diameter of the circle.



Since, centre is the midpoint of the diameter of the circle.

So coordinates of midpoint of diameter

$$AB = \left(\frac{8+x}{2}, \frac{10+y}{2}\right)$$

But O(4, 5) is the centre hence

$$\frac{8+x}{2} = 4 \Rightarrow x = 8 - 8 = 0.$$

Also $\quad \frac{10+y}{2} = 5 \Rightarrow y = 10 - 10 = 0.$

Hence (0, 0) be the coordinates of the other end.

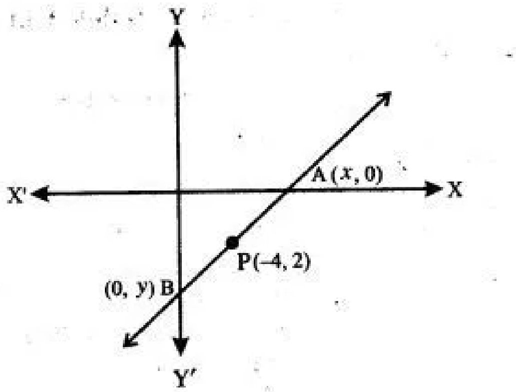
Question 5. In the following figure line APB meets the X-axis at A, Y-axis at B. P is the point (4, -2) and AP : PB = 1 : 2. Write down the coordinates of A and B.

Solution : Let $(x, 0)$ and $(0, y)$ be the coordiantes of A and B respectively.

Point P divides AB in the ratio of 1 : 2.

So coordinates of P

$$4 = \frac{1 \times 0 + 2 \times x}{1 + 2}$$



$$\Rightarrow \quad 2x = 4 \times 3$$

$$\Rightarrow \quad x = 6$$

Also $$-2 = \frac{2 \times 0 + 1 \times y}{1 + 2}$$

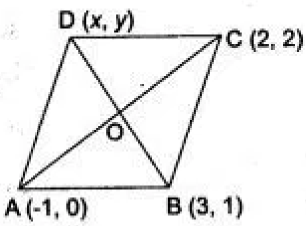
$$\Rightarrow \quad -6 = y$$

$$\Rightarrow \quad y = -6.$$

Hence, the coordinates of A and B are (– 6, 0) and (0, 6) respectively. Ans.

Question 6. The three vertices of a parallelogram taken in order are (-1, 0), (3, 1) and (2, 2) respectively. Find the coordinates of the fourth vertex.

Solution : Let A(–1, 0), B (3, 1), C (2, 2) and D (x, y) be the vertices of a parallelogram ABCD taken in order.



Since, the diagonals of a parallelogram bisect each other.

So, coordinates of the mid point of AC = coordinates of mid point of BD

$$\Rightarrow \quad \left(\frac{-1+2}{2}, \frac{0+2}{2}\right) = \left(\frac{3+x}{2}, \frac{y+1}{2}\right)$$

$$\Rightarrow \quad \left(\frac{1}{2}, 1\right) = \left(\frac{3+x}{2}, \frac{y+1}{2}\right)$$

$$\frac{3+x}{2} = \frac{1}{2} \Rightarrow \quad x = -2$$

Also $\quad \frac{y+1}{2} = 1 \Rightarrow y + 1 = 2$

$$\Rightarrow \quad y = 1$$

The fourth vertex of parallelogram = (– 2, 1).

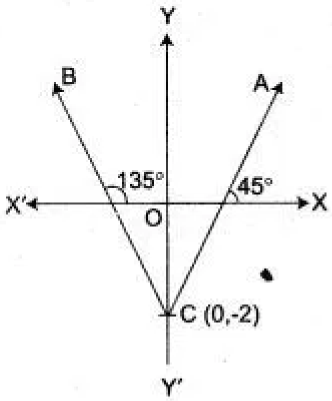
Question 7. Find the equation of a straight line which cuts an intercept – 2 units from Y-axis and being equally inclined to the axis.

Solution : Since, the required line is equally inclined with coordinate axis, therefore, it makes either an angle of 45° or 135° with the X-axis.

So, its slope is $\quad m = \tan 45° \Rightarrow m = 1$

or $\quad m = \tan 135° \Rightarrow m = -1$

Y-intercept, $c = -2$



Hence, the equation of required lines are

$$y = mx + c$$

i.e., $\quad y = 1 \cdot x - 2$ or $y = -1 \cdot x - 2$

$\Rightarrow \quad y = x - 2$ or $y = -x - 2$

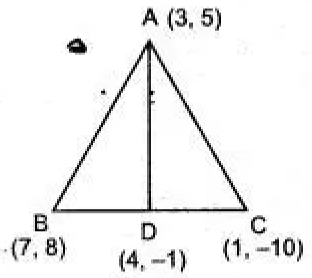
$\Rightarrow \quad x - y - 2 = 0$ or $x + y + 2 = 0.$ Ans.

Question 8. In ΔABC, A (3, 5), B (7, 8) and C (1, – 10). Find the equation of the median through A.

Solution : Coordinates of

$$D\left(\frac{7+1}{2}, \frac{8-10}{2}\right) = (4, -1)$$

(Midpoint formula)



Now equation of AD (Median through A)

$$y - 5 = \frac{-1-5}{4-3}(x-3)$$

(Two point form)

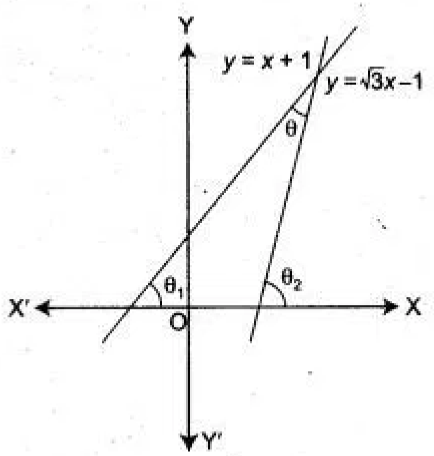
$$y - 5 = -6(x-3)$$

$$y - 5 = -6x + 18$$

or $6x + y - 23 = 0$. Ans.

Question 9. The figure alongside (not drawn to scale) represents the lines $y = x + 1$ and $y = \sqrt{3}x - 1$.

(i) Find the angle which the line $y = x + 1$ makes with X-axis.



(ii) Find the angle which the line $y = \sqrt{3}x - 1$ makes with X-axis.

(iii) Determine angle θ.

(iv) Find the point where the line $y = x + 1$ meets X-axis.

(v) Find the point where the line $y = \sqrt{3}x - 1$ meets Y-axis.

Solution : (i) $y = x + 1$

$\Rightarrow \quad m_1 = \tan\theta_1 = 1 = \tan 45°$

$\Rightarrow \quad \theta_1 = 45°.$ Ans.

(ii) $y = \sqrt{3}x - 1$

$\Rightarrow \quad m_2 = \tan\theta_2 = \sqrt{3} = \tan 60°$

$\Rightarrow \quad \theta_2 = 60°.$ Ans.

(iii) $\therefore \quad \theta = \theta_2 - \theta_1.$

($\because$ Exterior $\angle$ = Sum of interior opposite $\angle$s)

$= 60° - 45° = 15°.$ Ans.

(iv) Put $y = 0$ in $y = x + 1$, we get

$0 = x + 1$

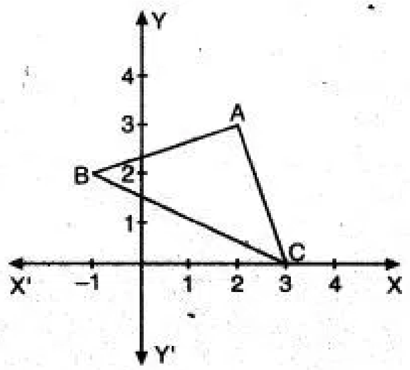
$\Rightarrow \quad x = -1$

$\therefore$ The required point is $(-1, 0)$. Ans.

(v) Put $x = 0$ in $y = \sqrt{3}x - 1$, we get $y = -1$

$\therefore$ The required point is $(0, -1)$. Ans.

Question 10. In the adjoining figure, write



(i) The coordinates of A, B and C.

(ii) The equation of the line through A and || to BC.

Solution : (i) A = (2, 3), B = (−1, 2), C = (3, 0). Ans.

(ii) Slope of BC $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-0}{-1-3} = \frac{2}{-4}$

$m_1 = \frac{-1}{2}$

Since lines are parallel

$\therefore \quad m_1 = m_2$

Hence, $\quad m_2 = -\frac{1}{2}$

and passing through point (2, 3)

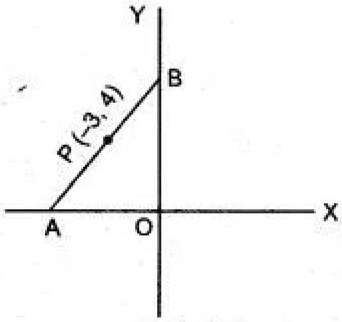
$\because$ Equation of line is $y - y_1 = m_2(x - x_1)$

$\therefore$ required line is $y - 3 = \frac{-1}{2}(x - 2)$

$2y - 6 = -x + 2$

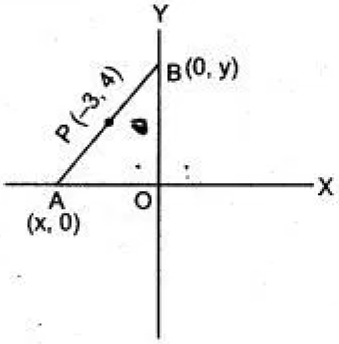
$x + 2y = 8.$ Ans.

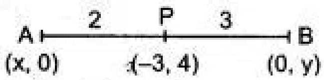
Question 11. In the figure given below, the line segment AB meets X-axis at A and Y-axis at B. The point P (- 3, 4) on AB divides it in the ratio 2 : 3. Find the coordinates of A and B.



Solution : AP : PB = 2 : 3

Let A $(x, 0)$ and B $(0, y)$





$\therefore$ By section formula

$$\frac{2 \times 0 + 3 \times x}{2 + 3} = -3$$

$\Rightarrow$ $3x = -15$

$x = -5$

and $\dfrac{2 \times y + 3 \times 0}{2 + 3} = 4$

$\Rightarrow$ $2y = 20$

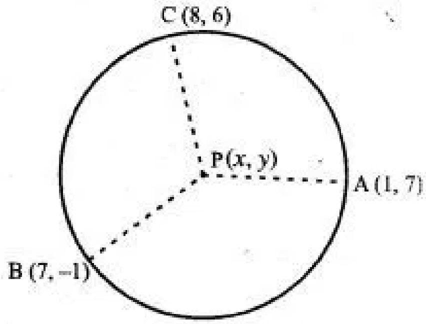
$\Rightarrow$ $y = 10$

$\therefore$ Coordinates of A $\equiv (x, 0) \equiv (-5, 0)$

and B $\equiv (0, y) \equiv (0, 10)$.

Question 12. Determine the centre of the circle on which the points (1, 7), (7 – 1), and (8, 6) lie. What is the radius of the circle ?

Solution : Let $P(x, y)$ be the centre of the circle and $A(1, 7)$, $B(7, -1)$ and $C(8, 6)$ be the given points.



Then $PA = PB = PC = \text{radius}$

$\Rightarrow \quad PA^2 = PB^2 = PC^2 = r^2$

$\Rightarrow \quad (x-1)^2 + (y-7)^2 = r^2 \quad \ldots(i)$

$(x-7)^2 + (y+1)^2 = r^2 \quad \ldots(ii)$

Also $\quad (x-8)^2 + (y-6)^2 = r^2 \quad \ldots(iii)$

Subtracting (ii) from (i)

$$(x^2 + 1 - 2x + y^2 + 49 - 14y) - (x^2 + 49 - 14x + y^2 + 1 + 2y) = 0$$

$\Rightarrow \quad 12x - 16y = 0$

$$y = \frac{3}{4}x \quad \ldots(iv)$$

Subtracting (iii) from (ii)

$$\Rightarrow (x^2 + 49 - 14x + y^2 + 1 + 2y) - (x^2 + 64 - 16x + y^2 + 36 - 12y) = 0$$

$\Rightarrow \quad 2x + 14y - 50 = 0$

$\Rightarrow \quad x + 7\left(\frac{3}{4}x\right) - 25 = 0$

$\Rightarrow \quad \frac{25x}{4} = 25$

$\Rightarrow \quad x = 4$

From (iv) $\quad y = \frac{3}{4}x = \frac{3}{4} \times 4$

$y = 3.$

The centre is $P(4, 3)$.

Also radius, $r = PA = \sqrt{(4-1)^2 + (3-7)^2}$

$= \sqrt{9+16} = 5$ units. Ans.

Question 13. Find the image of a point (-1, 2) in the line joining (2, 1) and (- 3, 2).

Solution : Let $D(\alpha, \beta)$ be the image of point $C(-1, 2)$ in the line joining the points $A(2, 1)$ and $B(-3, 2)$.

Since, AB is the perpendicular bisector of CD.

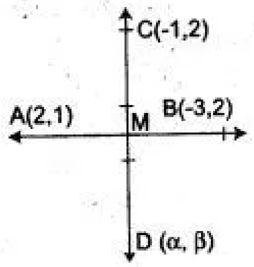
So, Slope of AB × Slope of CD = −1

$$\Rightarrow \quad \frac{2-1}{-3-2} \times \frac{\beta-2}{\alpha+1} = -1$$

$$\Rightarrow \quad \frac{1}{-5} \times \frac{\beta-2}{\alpha+1} = -1$$

$$\Rightarrow \quad \beta - 2 = 5\alpha + 5$$

$$\Rightarrow \quad 5\alpha - \beta + 7 = 0 \quad \text{...(i)}$$



Equation of line AB,

$$y - 1 = \frac{2-1}{-3-2}(x-2)$$

$$\Rightarrow \quad y - 1 = \frac{1}{-5}(x-2)$$

$$\Rightarrow \quad -5(y-1) = x - 2$$

$$\Rightarrow \quad x - 2 + 5y - 5 = 0$$

$$\Rightarrow \quad x + 5y - 7 = 0 \quad \text{...(ii)}$$

Since, midpoint M of CD $\left(\frac{\alpha-1}{2}, \frac{\beta+2}{2}\right)$ lies on AB.

$$\frac{\alpha-1}{2} + 5\left(\frac{\beta+2}{2}\right) - 7 = 0$$

$$\Rightarrow \quad \alpha - 1 + 5\beta + 10 - 14 = 0$$

$$\Rightarrow \quad \alpha + 5\beta - 5 = 0 \quad \text{...(iii)}$$

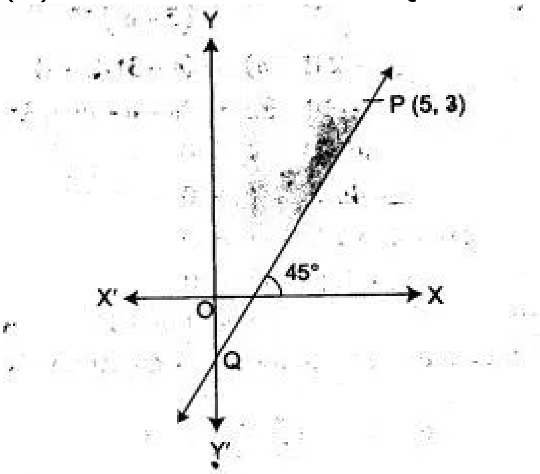
Solving (i) and (iii), we get

$$\alpha = \frac{-15}{13} \text{ and } \beta = \frac{16}{13}$$

Hence, coordinates of D are $\left(-\frac{15}{13}, \frac{16}{13}\right)$. Ans.

Question 14. The line through P (5, 3) intersects Y axis at Q.
(i) Write the slope of the line.
(ii) Write the equation of the line.

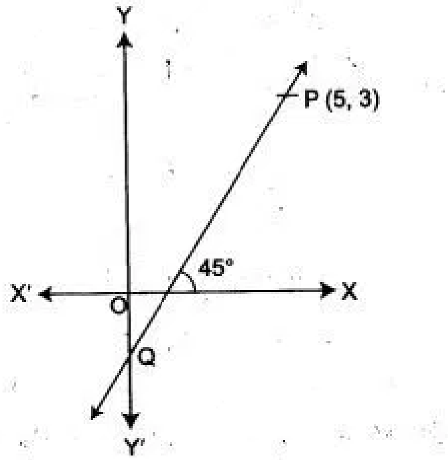
(iii) Find the coordinates of Q.



Solution :

(i)

$$m = \tan\theta = \tan 45°$$
$$m = 1$$



(ii) Equation of line PQ

$$y - y_1 = m(x - x_1)$$
$$y - 3 = 1(x - 5)$$
$$y - 3 = x - 5$$
$$\Rightarrow \quad x - y - 2 = 0$$

(iii) Equation of PQ is

$$x - y - 2 = 0$$

Put $x = 0$ (coordinates of Q)

$$-y - 2 = 0$$
$$\Rightarrow \quad y = -2$$

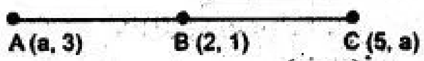
So, coordinates of Q $(0, -2)$.

Question 15. Find the value of 'a' for which the following points A (a, 3), B (2, 1) and C (5, a) are collinear. Hence find the equation of the line.

Solution : Equation of line passing through AC is

$$(y-3) = \left(\frac{a-3}{5-a}\right)(x-a)$$

As if A, B and C are callinear than B will satisfy it, *i.e.,*



$$(1-3) = \left(\frac{a-3}{5-a}\right)(2-a)$$

$$-2(5-a) = (a-3)(2-a)$$

$$-10+2a = 2a-6-a^2+3a$$

$$a^2-3a-4 = 0$$

$$a^2-4a+a-4 = 0$$

$$a(a-4)+1(a-4) = 0$$

$$(a-4)(a+1) = 0$$

$\Rightarrow$ $a = 4 \text{ or } -1.$ **Ans.**

Thus, required equation of straight line is

$$(y-3) = \left(\frac{4-3}{5-4}\right)(x-4)$$

$$y-3 = \left(\frac{1}{1}\right)(x-4)$$

$$x-y-1 = 0$$

or $$(y-3) = \left(\frac{-1-3}{5+1}\right)(x+1)$$

$$(y-3) = \left(\frac{-4}{6}\right)(x+1)$$

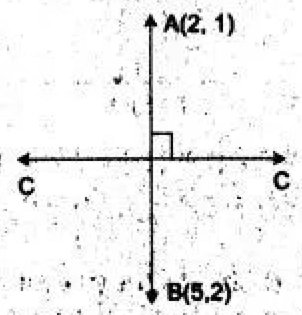
$$y-3 = \frac{-2}{3}(x+1)$$

$$3y-9 = -2x-2$$

$$2x+3y-7 = 0,$$ **Ans.**

Question 16. If the image of the point (2,1) with respect to the line mirror be (5, 2). Find the equation of the mirror.

Solution : Let CD be the line mirror with slope m_1.



Now the slope of the line joining A(2, 1) and B(5, 2).

$$m_2 = \frac{2-1}{5-2} = \frac{1}{3}$$

Since $\quad CD \perp AB$

So, $\quad m_1 m_2 = -1$

$\Rightarrow \quad m_1 \times \frac{1}{3} = -1$

$\Rightarrow \quad m_1 = -3.$

Now mid point of $AB = \left(\frac{2+5}{2}, \frac{1+2}{2}\right) = \left(\frac{7}{2}, \frac{3}{2}\right)$

Equation of the mirror CD,

$$y - y_1 = m(x - x_1)$$

$\Rightarrow \quad y - \frac{3}{2} = -3\left(x - \frac{7}{2}\right)$

$\Rightarrow \quad y - \frac{3}{2} = -3x + \frac{21}{2}$

$\Rightarrow \quad 2y - 3 = -6x + 21$

$\Rightarrow \quad 6x + 2y - 3 - 21 = 0$

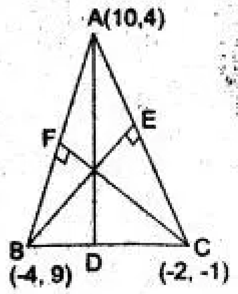
$\Rightarrow \quad 6x + 2y - 24 = 0$

or $\quad 3x + y - 12 = 0.$ Ans.

Question 17. The vertices of a triangle are A(10, 4), B(- 4, 9) and C(- 2, -1). Find the

Solution : Let AD, BE and CF be the three altitudes of ΔABC then

$$AD \perp BC$$
$$BE \perp CA$$
and
$$CF \perp AB.$$



$$\text{Slope of BC} = \frac{-1-9}{-2+4} = -5$$

Since $AD \perp BC$

Slope of BC × slope of AD = − 1

$$\text{Slope of AD} = \frac{-1}{-5} = \frac{1}{5}$$

Therefore $AD \perp BC$

Since, AD passes through $A(10, 4)$

So, equation of AD is

$$y - y_1 = m(x - x_1)$$
$$y - 4 = \frac{1}{5}(x - 10)$$

$$\Rightarrow \quad 5y - 20 = x - 10$$
$$\Rightarrow \quad x - 5y + 10 = 0 \qquad \ldots(i)$$

$$\text{Now, Slope of AC} = \frac{4+1}{10+2} = \frac{-5}{12}$$

Since $BE \perp AC$

Slope of BE × Slope of AC = − 1

$$\text{So, Slope of BE} = \frac{-1 \times 12}{5} = -\frac{12}{5}.$$

Equation of BE which passes through B(– 4, 9) is

$$y - y_1 = m(x - x_1)$$
$$y - 9 = -\frac{12}{5}(x + 4)$$

or $12x + 5y + 3 = 0$ ...(ii)

Now Slope of $AB = \frac{4-9}{10+4} = \frac{-5}{14}$

Since CF $\perp$ AB.

So,

$$\text{Slope of AB} \times \text{Slope of CF} = -1$$
$$\Rightarrow -\frac{5}{14} \times \text{Slope of CF} = -1$$
$$\Rightarrow \text{Slope of CF} = \frac{14}{5}$$

Equation of CF which passes through C(–2, –1) is

$$y - y_1 = m(x - x_1)$$
$$y + 1 = \frac{14}{5}(x + 2)$$
$$\Rightarrow 14x - 5y + 23 = 0 \quad \text{...(iii)}$$

Thus, the equation of altitudes of ΔABC are

$$x - 5y + 10 = 0$$
$$12x + 5y + 3 = 0$$

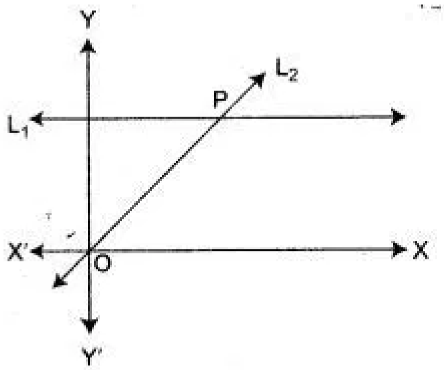
and $14x - 5y + 23 = 0.$ Ans.

Question 18. Given equation of line L_1 is y = 4.

(i) Write the slope of line, if L_2 is the bisector of angle O.

(ii) Write the coordinates of point P.

(iii) Find the equation of L_2

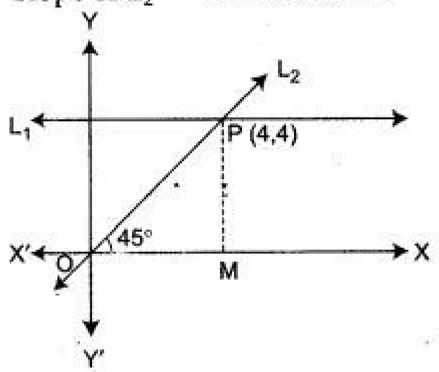


Solution : Equation of L_1 is $y = 4$ (given)

(i) As L_2 is bisector of O

$\Rightarrow L_2$ is inclined at an angle of 45° with XX′

$\therefore$ Slope of $L_2 = m = \tan 45° = 1$



(ii) Slope of $L_2 = \frac{4-0}{x-0} \Rightarrow 1 = \frac{4}{x} \Rightarrow x = 4$

So coordinates of P are (4, 4).

(Since the slope of L_2 is 1, $L_2 \Rightarrow PM = OM$)

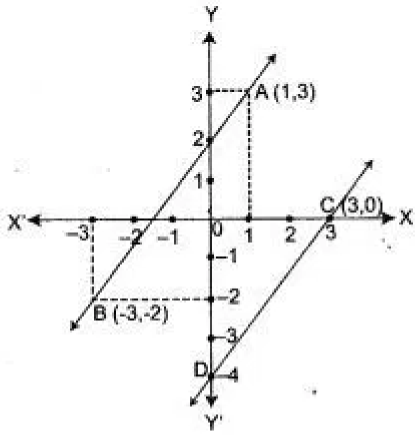
(iii) L_2 passes through O (0, 0), P (4, 4) and has slope $m = 1$

$\therefore$ Equation of L_2 is

$$y - y_1 = m(x - x_1)$$
$$y - 0 = 1(x - 0)$$
or $$y = x$$
or $$x - y = 0.$$ Ans.

Question 19. From the adjacent figure:

(i) Write the coordinates of the points A, B, and



(ii) Write the slope of the line AB.

(iii) Line through C, drawn parallel to AB, intersects Y-axis at D. Calculate the co-ordintes of D.

Solution : (i) Coordinates of the points A, B and C are (1, 3), (– 3, – 2) and (3, 0) respectively.

(ii) Slope of AB $= \frac{-2-3}{-3-1} = \frac{5}{4}$.

(iii) Line through C(3, 0) and parallel to AB.

$\therefore$ Slope $= \frac{5}{4}$.

$\therefore$ Equation to the line is

$$y - y_1 = m(x - x_1)$$
$$y - 0 = \frac{5}{4}(x - 3)$$
$$4y = 5x - 15$$

This line intersects Y-axis at D.

$\therefore$ On solving

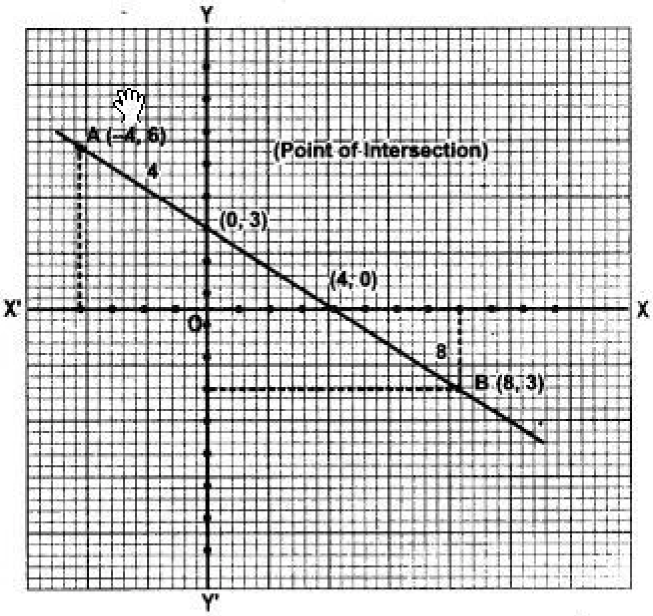
$$4y = 5x - 15$$
and $$x = 0, \quad \text{(Equation to Y-axis)}$$
We get, $$4y = -15$$
$$y = -\frac{15}{4}$$

$\therefore$ Coordinates of point D are $\left(0, \frac{-15}{4}\right)$. Ans.

Graphical Depiction

Question 1. Given a line segment AB joining the points A (- 4, 6) and B (8, – 3). Find:
(i) the ratio in which AB is divided by the y- axis.
(ii) find the ordinates of the point of intersection.
(iii) the length of AB.

Solution :



A (– 4, 6), B (8, – 3)

Let ratio be $k : 1$

(i) Where the y-axis divide AB, $x = 0$

$$\therefore \quad x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}$$

$$\therefore \quad 0 = \frac{k \times 8 + 1 \times (-4)}{k+1}$$

$$\Rightarrow \quad 8k - 4 = 0$$

$$\Rightarrow \quad 8k = 4$$

$$\Rightarrow \quad k = \frac{4}{8} = \frac{1}{2}$$

So, ratio be 1 : 2

(ii) Now, $$y = \frac{1 \times (-3) + 2 \times 6}{1+2}$$

$$= \frac{-3+12}{3} = 3$$

So, point of intersection (0, 3)

(iii) $$AB = \sqrt{(8+4)^2 + (-3-6)^2}$$

$$= \sqrt{(12)^2 + (-9)^2}$$

$$= \sqrt{144 + 81} = \sqrt{225}$$

$$= 15 \text{ units.}$$