JEE (Main)-2025 (Online) Session-2

Question Paper with Solutions

(Mathematics, Physics, And Chemistry)

4 April 2025 Shift – 1

Time: 3 hrs. **M.M : 300**

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) This test paper consists of 75 questions. Each subject (PCM) has 25 questions. The maximum marks are 300.

(3) This question paper contains Three Parts. Part-A is Physics, Part-B is Chemistry and Part-C is Mathematics. Each part has only two sections: Section-A and Section-B.

(4) Section - A : Attempt all questions.

(5) Section - B : Attempt all questions.

(6) Section - A (01 - 20) contains 20 multiple choice questions which have only one correct answer. Each question carries +4 marks for correct answer and -1 mark for wrong answer.

(7) Section - B (21 – 25) contains 5 Numerical value based questions. The answer to each question should be rounded off to the nearest integer. Each question carries +4 marks for correct answer and -1 mark for wrong answer.

SECTION-A

1. Let f, g: $(1, \infty) \to \mathbb{R}$ be defined as $f(x) = \frac{2x+3}{5x+2}$ and $g(x) = \frac{2-3x}{1-x}$. If the range of the function $fog : [2, 4] \to \mathbb{R}$ is $[\alpha, \beta]$, then $\frac{1}{\beta - \alpha}$ is equal to

(1) 68 (2) 29

(3) 2 (4) 56

Ans. **(4)**

Sol. fog (x) = f(g(x))

$$= f\left(\frac{2-3x}{1-x}\right) = \frac{2\left(\frac{2-3x}{1-x}\right)+3}{5\left(\frac{2-3x}{1-x}\right)+2}$$

$$= \frac{4-6x+3-3x}{10-15x+2-2x} = \left(\frac{7-9x}{12-17x}\right)$$

$$\therefore \left[\begin{array}{l} 12-7x \neq 0 \\ x \neq \frac{12}{17} \end{array}\right.$$

$$\left[\begin{array}{l} fog(2) = \frac{7-9(2)}{12-17(2)} = \frac{-11}{-22} = \frac{1}{2} \\ fog(4) = \frac{7-9(4)}{12-17(4)} = \frac{-29}{-56} = \frac{29}{56} \end{array}\right.$$

Range of fog : $[\alpha, \beta] = \left[\frac{1}{2}, \frac{29}{56}\right]$

$$\therefore (\beta - \alpha) = \frac{29}{56} - \frac{1}{2} = \frac{29-28}{56} = \frac{1}{56}$$

$$\frac{1}{(\beta-\alpha)} = 56$$

2. Consider the sets $A = \{(x, y) \in \mathbb{R} \times \mathbb{R} : x^2 + y^2 = 25\}$, $B = \{(x, y) \in \mathbb{R} \times \mathbb{R} : x^2 + 9y^2 = 144\}$, $C = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} : x^2 + y^2 \leq 4\}$, and $D = A \cap B$. The total number of one-one functions from the set D to the set C is:

(1) 15120 (2) 19320

(3) 17160 (4) 18290

Ans. **(3)**

Sol. $A : x^2 + y^2 = 25$ ....(1)

$B : \frac{x^2}{144} + \frac{y^2}{16} = 1$ ....(2)

$C : x^2 + y^2 \leq 4$ ....(3)

Solve (1) & (2)

$x^2 + 9(25 - x^2) = 144$

$-8x^2 = 144 - 225 = -81$

$x = \pm \frac{9}{2\sqrt{2}}$

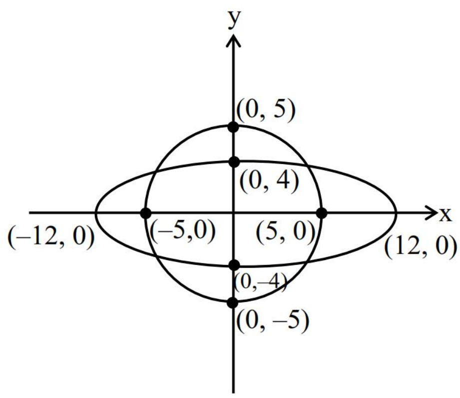
By (1) $\Rightarrow y = \pm\sqrt{25 - x^2}$

$= \pm\sqrt{25 - \frac{81}{8}} = \pm\frac{\sqrt{119}}{2\sqrt{2}}$

$\therefore D = A \cap B =$

$$\left\{\left(\frac{9}{2\sqrt{2}}, \frac{\sqrt{119}}{2\sqrt{2}}\right), \left(\frac{9}{2\sqrt{2}}, -\frac{\sqrt{119}}{2\sqrt{2}}\right), \left(\frac{-9}{2\sqrt{2}}, \frac{\sqrt{119}}{2\sqrt{2}}\right), \left(\frac{-9}{2\sqrt{2}}, \frac{-\sqrt{119}}{2\sqrt{2}}\right)\right\}$$

No. of elements in set D = 4



$\because C = \{(x, y) \in Z \times Z : x^2 + y^2 \leq 4\}$

= {(0, 2), (2, 0), (0, –2), (–2, 0), (1, 1), (–1, –1), (1, –1), (–1, 1), (1, 0), (0, 1), (–1, 0), (0, –1), (0, 0)}

No. of elements in set C = 13

Total no. of one-one function from

Set D to sec C $\Rightarrow 13 \times 12 \times 11 \times 10 = 17160$

3. Let A = {1, 6, 11, 16, …} and B = {9, 16, 23, 30, …} be the sets consisting of the first 2025 terms of two arithmetic progressions. Then n $(A \cup B)$ is

(1) 3814 (2) 4027

(3) 3761 (4) 4003

Ans. (3)

Sol. A = {1, 6, 11, 16, 21, 26, 31, 36, 41, 46, 51, 56, 61, 66, 71, 76, 81, 86, 91, ……}

B = {9, 16, 23, 30, 37, 44, 51, 58, 65, 72, 79, 86, 93, 100, ……}

$A \cap B$ = {16, 51, 86, …..}

For set 'A' $\Rightarrow T_{2025} = 1 + (2025 - 1)(5) = 10121$

For set 'B' $\Rightarrow T_{2025} = 9 + (2025 - 1)(7) = 14177$

So, for $(A \cap B) \Rightarrow T_n = 16 + (n - 1)(35) \le 10121$

$$(n-1) \le \frac{10121 - 16}{35} = 288.71$$

$n \le 289.71 \Rightarrow n = 289$

$\therefore n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 2025 + 2025 - 289 = 3761$

4. For an integer $n \ge 2$, if the arithmetic mean of all coefficients in the binomial expansion of $(x + y)^{2n-3}$ is 16, then the distance of the point $P(2n - 1, n^2 - 4n)$ from the line x + y = 8 is:

(1) $\sqrt{2}$ (2) $2\sqrt{2}$

(3) $5\sqrt{2}$ (4) $3\sqrt{2}$

Ans. (4)

Sol. No. of terms in $(x + y)^{(2n-3)} \Rightarrow \left[\begin{array}{l}(2n-3+1)\\ (2n-2)\end{array}\right.$

$\therefore$ sum of all coefficients $= 2^{2n-3}$

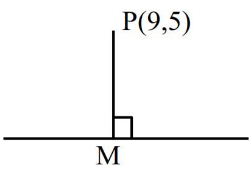
(Put x = y = 1)

$\therefore$ Arithmetic mean of all coefficients

$$= \left(\frac{2^{2n-3}}{2n-2}\right) = 16$$

$\Rightarrow 2^{2n-3} = 2^5(n - 1) \Rightarrow n = 5$

$\therefore P(2n - 1, n^2 - 4n) = (9, 5)$



x + y = 8

$$\therefore PM = \left|\frac{9+5-8}{\sqrt{2}}\right| = \frac{6}{\sqrt{2}} = \frac{3\times 2}{\sqrt{2}} = 3\sqrt{2}$$

5. The probability, of forming a 12 persons committee from 4 engineers, 2 doctors and 10 professors containing at least 3 engineers and at least 1 doctor, is:

(1) $\frac{129}{182}$ (2) $\frac{103}{182}$

(3) $\frac{17}{26}$ (4) $\frac{19}{26}$

Ans. (1)

Sol. 3 engineering + 1 doctor + 8 Prof $\rightarrow {}^4C_3 . {}^2C_1 . {}^{10}C_8$

= 360

3 engineering + 2 doctors + 7 Prof $\rightarrow {}^4C_3 . {}^2C_2 . {}^{10}C_7$

= 480

4 engineering + 1 doctor + 7 Prof $\rightarrow {}^4C_4 . {}^2C_1 . {}^{10}C_7$

= 240

4 engineering + 2 doctors + 6 Prof $\rightarrow {}^4C_4 . {}^2C_2 . {}^{10}C_6$

= 210

Total = 1290

$$\text{Req. probability} = \frac{1290}{{}^{16}C_{12}} = \frac{1290}{1820} = \frac{129}{182}$$

Ans. (1)

6. Let the shortest distance between the lines $\frac{x-3}{3} = \frac{y-\alpha}{-1} = \frac{z-3}{1}$ and $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-\beta}{4}$ be $3\sqrt{30}$. Then the positive value of $5\alpha + \beta$ is

(1) 42 (2) 46

(3) 48 (4) 40

Ans. (2)

Sol. $A(3, \alpha, 3)$ & $B(-3, -7, \beta)$

$\overrightarrow{BA} = 6\hat{i} + (\alpha + 7)\hat{j} + (3 - \beta)\hat{k}$

$$\vec{p} \times \vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 1 \\ -3 & 2 & 4 \end{vmatrix}$$

$$\frac{\left|\overrightarrow{BA}.(\vec{p}\times\vec{q})\right|}{\left|\vec{p}\times\vec{q}\right|} = 3\sqrt{30}$$

$36 + 15(\alpha + 7) - 3(3 - \beta) = \left(3\sqrt{30}\right)^2$

$36 + 15\alpha + 105 - 9 + 3\beta = 270$

$15\alpha + 3\beta = 138$

$5\alpha + \beta = 46$

7. If $\lim_{x\to 1^+}\frac{(x-1)(6+\lambda\cos(x-1))+\mu\sin(1-x)}{(x-1)^3}=-1$, where $\lambda, \mu \in \mathbb{R}$, then $\lambda + \mu$ is equal to

(1) 18 (2) 20

(3) 19 (4) 17

Ans. (1)

Sol. Put $x = 1 + h$

$$\lim_{h\to 0}\frac{h(6+\lambda\cos h)-\mu\sin h}{h^3}=-1$$

$$\lim_{h\to 0}\frac{h\left(6+\lambda\left(1-\frac{h^2}{2!}\right)\right)-\mu\left(h-\frac{h^3}{3!}\right)}{h^3}=-1$$

$$6+\lambda-\mu=0 \text{ and } -\frac{\lambda}{2}+\frac{\mu}{6}=-1$$

$$\lambda+\mu=18$$

8. Let $f : [0, \infty) \to \mathbb{R}$ be differentiable function such that $f(x) = 1 - 2x + \int_0^x e^{x-t} f(t)\,dt$ for all $x \in [0, \infty)$.

Then the area of the region bounded by $y = f(x)$ and the coordinate axes is

(1) $\sqrt{5}$ (2) $\frac{1}{2}$

(3) $\sqrt{2}$ (4) 2

Ans. (2)

Sol. $y = 1 - 2x + e^x\int_0^x e^{-t}f(t)dt$

$$\frac{dy}{dx}=-2+e^{-x}\cdot e^x f(x)+e^x\int_0^x e^{-t}f(t)dt$$

$$\frac{dy}{dx}=-2+y+y+2x-1$$

$$\frac{dy}{dx}-2y=(2x-3)$$

$$ye^{-2x}=\int(2x-3)dx\cdot e^{-2x}$$

$$ye^{-2x}=\frac{-(2x-3)}{2}e^{-2x}+\int e^{-2x}dx$$

$$ye^{-2x}=\frac{-(2x-3)}{2}e^{-2x}-\frac{1}{2}e^{-2x}+c$$

$$f(0)=1\Rightarrow c=1-\frac{3}{2}+\frac{1}{2}=0$$

$$y=-\frac{(2x-3)}{2}-\frac{1}{2}$$

$$y=-x+1$$

$$x+y=1$$

$$\text{area}=\frac{1}{2}(1)(1)=\frac{1}{2}$$

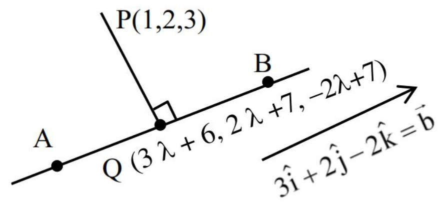
9. Let A and B be two distinct points on the line $L:\frac{x-6}{3}=\frac{y-7}{2}=\frac{z-7}{-2}$. Both A and B are at a distance $2\sqrt{17}$ from the foot of perpendicular drawn from the point (1, 2, 3) on the line L. If O is the origin, then $\overrightarrow{OA}\cdot\overrightarrow{OB}$ is equal to:

(1) 49 (2) 47

(3) 21 (4) 62

Ans. (2)

Sol.



$$\overrightarrow{PQ}.\vec{b}=0$$

$$\Rightarrow 3(3\lambda+5)+2(2\lambda+5)-2(-2\lambda+4)$$

$$\Rightarrow 17\lambda=-17\Rightarrow \boxed{\lambda=-1}$$

Q (3,5,9)

Let A $(3\mu + 6, 2\mu + 7, -2\mu + 7)$

$$(3\mu+3)^2+(2\mu+2)^2+(-2\mu-2)^2=68$$

$$\Rightarrow \mu^2+2\mu-3=0 \quad \mu=-3 \text{ or } \mu=1$$

A (–3, 1, 13) and B (9,9,5)

$$\overrightarrow{OA}.\overrightarrow{OB}=-27+9+65=47$$

10. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function satisfying $f(0) = 1$ and $f(2x) - f(x) = x$ for all $x \in \mathbb{R}$. If $\lim_{n\to\infty}\left\{f(x)-f\left(\frac{x}{2^n}\right)\right\}=G(x)$, then $\sum_{r=1}^{10}G(r^2)$ is equal to

(1) 540 (2) 385

(3) 420 (4) 215

Ans. (2)

Sol. $f(2x) - f(x) = x$

$$f(x) - f\left(\frac{x}{2}\right) = \frac{x}{2}$$

$$f\left(\frac{x}{2}\right) - f\left(\frac{x}{4}\right) = \frac{x}{4}$$

$$f\left(\frac{x}{4}\right) - f\left(\frac{x}{8}\right) = \frac{x}{8}$$

$$\vdots$$

$$f\left(\frac{x}{2^{n-1}}\right) - f\left(\frac{x}{2^n}\right) = \frac{x}{2^n}$$

$$f(2x) - f\left(\frac{x}{2^n}\right) = x\left\{\frac{1-\left(\frac{1}{2}\right)^{n-1}}{1-\frac{1}{2}}\right\}$$

$$f(x) - f\left(\frac{x}{2^n}\right) = 2x\left(1-\left(\frac{1}{2}\right)^{n+1}\right)$$

$$f(x) + x - f\left(\frac{x}{2^n}\right) = 2x\left(1-\left(\frac{1}{2}\right)^{n+1}\right)$$

$$\lim_{n\to\infty}\left(f(x) - f\left(\frac{x}{2^n}\right)\right) = \lim_{n\to\infty}\left(2x\left(1-\left(\frac{1}{2}\right)^{n+1}\right) - x\right)$$

$$G(x) = x$$

$$\sum_{r=1}^{10} G(r^2) = \sum_{r=1}^{10} r^2 = 385$$

11. $1 + 3 + 5^2 + 7 + 9^2 + \ldots$ upto 40 terms is equal to

(1) 43890 (2) 41880

(3) 33980 (4) 40870

Ans. **(2)**

Sol. $(1^2 + 5^2 + 9^2 + \ldots$...upto 20 terms$) + (3 + 7 + 11 + \ldots$...upto 20 terms$)$

$$= \sum_{r=1}^{20}(4r-3)^2 + \sum_{r=1}^{20}(4r-1)$$

$$= \sum_{r=1}^{20}(4r-3)^2 + (4r-1)$$

$$= 4\sum_{r=1}^{20}(4r^2 - 5r + 2)$$

$$= 16\sum_{r=1}^{20} r^2 - 20\sum_{r=1}^{20} r + 8\sum_{r=1}^{20} 1 = 41880$$

12. In the expansion of $\left(\sqrt[3]{2} + \frac{1}{\sqrt[3]{3}}\right)^n$, $n \in N$, if the ratio of 15^{th} term from the beginning to the 15^{th} term from the end is $\frac{1}{6}$, then the value of ${}^nC_3$ is:

(1) 4060 (2) 1040

(3) 2300 (4) 4960

Ans. **(3)**

Sol. $T_{r+1} = {}^nC_r(2^{1/3})^{n-r}\left(\frac{1}{3^{1/3}}\right)^r$

$r = 14$

$$T_{15} = {}^nC_{14}(2^{1/3})^{n-14}\left(\frac{1}{3^{1/3}}\right)^{14}$$

$T'_{15} = 15^{th}$ term from last is $(n-13)^{th}$ term from beginning.

$$T'_{15} = {}^nC_{n-14}(2^{1/3})^{14}\left(\frac{1}{3^{1/3}}\right)^{n-14}$$

$$\Rightarrow \frac{T_{15}}{T'_{15}} = \frac{{}^nC_{14}(2^{1/3})^{n-14}\left(\frac{1}{3^{1/3}}\right)^{14}}{{}^nC_{n-14}(2^{1/3})^{14}\left(\frac{1}{3^{1/3}}\right)^{n-14}} = \frac{1}{6}$$

$$= (2^{1/3})^{n-28}(3^{1/3})^{n-28} = \frac{1}{6}$$

$$= 6^{\frac{n-28}{3}} = 6^{-1}$$

$= n = 25$

So, ${}^nC_3 = {}^{25}C_3 = 2300$

13. Considering the principal values of the inverse trigonometric functions,

$\sin^{-1}\left(\frac{\sqrt{3}}{2}x + \frac{1}{2}\sqrt{1-x^2}\right), -\frac{1}{2} < x < \frac{1}{\sqrt{2}}$, is equal to

(1) $\frac{\pi}{4} + \sin^{-1}x$ (2) $\frac{\pi}{6} + \sin^{-1}x$

(3) $\frac{-5\pi}{6} - \sin^{-1}x$ (4) $\frac{5\pi}{6} - \sin^{-1}x$

Ans. **(2)**

Sol. $\sin^{-1}\left(\frac{\sqrt{3}}{2}x+\frac{1}{2}\sqrt{1-x^2}\right),\ \frac{-1}{2}<x<\frac{1}{\sqrt{2}}$

$\Rightarrow$ Let $\sin^{-1}(x)=\theta \qquad \frac{-\pi}{6}<\theta<\frac{\pi}{4}$

$\Rightarrow x=\sin\theta$, then

$\Rightarrow \sin^{-1}\left(\frac{\sqrt{3}}{2}\sin\theta+\frac{1}{2}\cos\theta\right)$

$\Rightarrow \sin^{-1}\left(\sin\left(\theta+\frac{\pi}{6}\right)\right)=\theta+\frac{\pi}{6}$

$\Rightarrow \sin^{-1}(x)+\frac{\pi}{6}$

14. Consider two vectors $\vec{u}=3\hat{i}-\hat{j}$ and $\vec{v}=2\hat{i}+\hat{j}-\lambda\hat{k}$, $\lambda>0$. The angle between them is given by $\cos^{-1}\left(\frac{\sqrt{5}}{2\sqrt{7}}\right)$. Let $\vec{v}=\vec{v}_1+\vec{v}_2$, where $\vec{v}_1$ is parallel to $\vec{u}$ and $\vec{v}_2$ is perpendicular to $\vec{u}$. Then the value $|\vec{v}_1|^2+|\vec{v}_2|^2$ is equal to

(1) $\frac{23}{2}$ (2) 14

(3) $\frac{25}{2}$ (4) 10

Ans. (2)

Sol. $\vec{u}=3\hat{i}-\hat{j},\ \vec{v}=2\hat{i}+\hat{j}-\lambda\hat{k}$,

$\Rightarrow \frac{\vec{u}.\vec{v}}{|\vec{u}||\vec{v}|}=\cos\theta$

$\Rightarrow \frac{5}{\sqrt{10}\sqrt{5+\lambda^2}}=\frac{\sqrt{5}}{2\sqrt{7}}$

$\Rightarrow \lambda^2=9 \quad \Rightarrow \lambda=3\ (\because \lambda>0)$

$\vec{v}=\vec{v}_1+\vec{v}_2$

$\Rightarrow |\vec{v}|^2=\vec{v}_1^2+\vec{v}_2^2+2\,\vec{v}_1\cdot\vec{v}_2$

$\Rightarrow 14=\vec{v}_1^2+\vec{v}_2^2+0 \qquad (\because \vec{v}_1\perp\vec{v}_2)$

$\Rightarrow |\vec{v}_1^2|+|\vec{v}_1^2|=14$

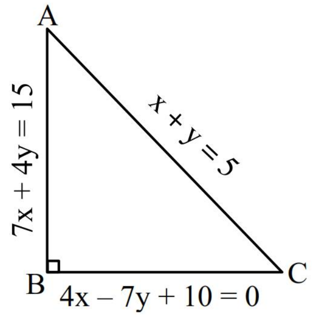
15. Let the three sides of a triangle are on the lines $4x-7y+10=0$, $x+y=5$ and $7x+4y=15$. Then the distance of its orthocentre from the orthocentre of the triangle formed by the lines $x=0$, $y=0$ and $x+y=1$ is

(1) 5 (2) $\sqrt{5}$

(3) $\sqrt{20}$ (4) 20

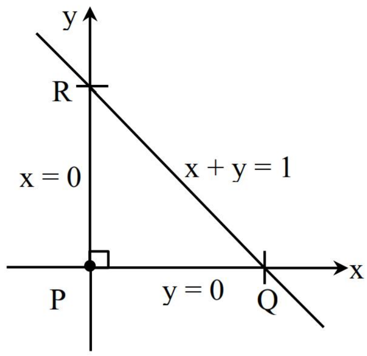
Ans. (2)

Sol.



Since triangle is right angle so orthocentre will be at right angle vertex

$7x+4y=15$
$7x+7y+10=0$ P.O.I B≡(1,2)



distance between P and B = $\sqrt{5}$

16. The value of $\int_{-1}^{1}\frac{\left(1+\sqrt{|x|-x}\right)e^x+\left(\sqrt{|x|-x}\right)e^{-x}}{e^x+e^{-x}}\,dx$ is equal to

(1) $3-\frac{2\sqrt{2}}{3}$ (2) $2+\frac{2\sqrt{2}}{3}$

(3) $1-\frac{2\sqrt{2}}{3}$ (4) $1+\frac{2\sqrt{2}}{3}$

Ans. (4)

Sol. $I = \int_{-1}^{1} \frac{\left(1+\sqrt{|-x|-(-x)}\right)e^{-x} + \left(\sqrt{|-x|-(-x)}\right)e^{-(-x)}}{e^{-x}+e^{-(-x)}} dx$

$\Rightarrow I = \int_{-1}^{1} \frac{\left(1+\sqrt{|x|+x}\right)e^{-x} + \left(\sqrt{|x|+x}\right)e^{x}}{e^{x}+e^{x}} dx$

$\Rightarrow 2I = \int_{-1}^{1} \frac{\left(1+\sqrt{|x|+x}+\sqrt{|x|-x}\right)\left(e^{x}+e^{-x}\right)}{\left(e^{x}+e^{-x}\right)} dx$

$\Rightarrow 2I = \int_{-1}^{1} \left(1+\sqrt{|x|+x}+\sqrt{|x|-x}\right) dx$

$\Rightarrow 2I = 2\int_{0}^{1} \left(1+\sqrt{|x|+x}+\sqrt{|x|-x}\right) dx$

$\Rightarrow 2I = 2\int_{0}^{1} \left(1+\sqrt{2x}+\sqrt{0}\right) dx$

$\Rightarrow I = \int_{0}^{1} \left(1+\sqrt{2x}\right) dx = \left[x + \frac{2\sqrt{2}}{3}x^{3/2}\right]_0^1$

$\Rightarrow I = \frac{2\sqrt{2}}{3} + 1$

17. The length of the latus-rectum of the ellipse, whose foci are (2, 5) and (2, –3) and eccentricity is $\frac{4}{5}$, is

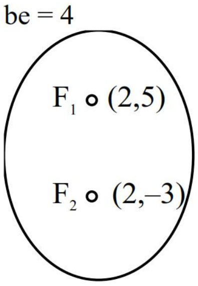
(1) $\frac{6}{5}$ (2) $\frac{50}{3}$

(3) $\frac{10}{3}$ (4) $\frac{18}{5}$

Ans. (4)

Sol. $2be = 8$

$be = 4$



$b\left(\frac{4}{5}\right) = 4 \Rightarrow b = 5$

$\because c^2 = b^2 - a^2$

$16 = 25 - a^2 \Rightarrow a = 3$

L.R. = $\frac{2a^2}{b} = \frac{18}{5}$

Option (4)

18. Consider the equation $x^2 + 4x - n = 0$, where $n \in [20, 100]$ is a natural number. Then the number of all distinct values of n, for which the given equation has integral roots, is equal to

(1) 7 (2) 8

(3) 6 (4) 5

Ans. (3)

Sol. $x^2 + 4x + 4 = n + 4$

$(x + 2)^2 = n + 4$

$x = -2 \pm \sqrt{n+4}$

$\because 20 \le n \le 100$

$\sqrt{24} \le \sqrt{n+4} \le \sqrt{104}$

$\Rightarrow \sqrt{n+4} \in \{5,6,7,8,9,10\}$

$\therefore$ '6' integral values of 'n' are possible

19. A box contains 10 pens of which 3 are defective. A sample of 2 pens is drawn at random and let X denote the number of defective pens. Then the variance of X is

(1) $\frac{11}{15}$ (2) $\frac{28}{75}$

(3) $\frac{2}{15}$ (4) $\frac{3}{5}$

Ans. (2)

Sol.

x	x = 0	x = 1	x = 2
P(x)	$\frac{^7C_2}{^{10}C_2}$	$\frac{^7C_1\,^3C_1}{^{10}C_2}$	$\frac{^3C_2}{^{10}C_2}$

$\mu = \Sigma x_i P(x_i) = 0 + \frac{7}{15} + \frac{2}{15} = \frac{3}{5}$

Variance $(x) = \Sigma P_i(x_i - \mu)^2 = \frac{28}{75}$

20. If $10\sin^4\theta + 15\cos^4\theta = 6$, then the value of $\frac{27\text{cosec}^6\theta + 8\sec^6\theta}{16\sec^8\theta}$ is:

(1) $\frac{2}{5}$ (2) $\frac{3}{4}$

(3) $\frac{3}{5}$ (4) $\frac{1}{5}$

Ans. (1)

Sol. $10(\sin^2\theta)^2 + 15(1 - \sin^2\theta)^2 = 6$

Let $\sin^2\theta = t \Rightarrow 10t^2 + 15(1-t)^2 = 16$

$10t^2 + 15 - 30t + 15t^2 = 6$

$25t^2 - 30t + 9 = 0$

$(5t - 3)^2 = 0$

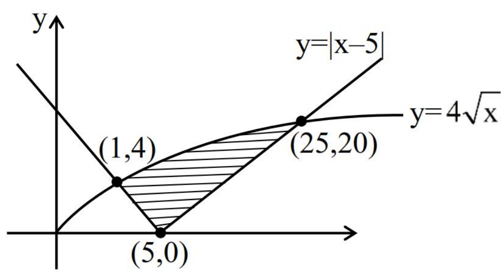
$\sin^2\theta = \frac{3}{5}$ and $\cos^2\theta = \frac{2}{5}$

$$\frac{27\times\frac{125}{27}+8+\frac{125}{8}}{16\left(\frac{5}{2}\right)^4} = \frac{250}{125\times 5} = \frac{2}{5}$$

SECTION-B

21. If the area of the region $\{(x, y) : |x-5| \le y \le 4\sqrt{x}\}$ is A, then 3A is equal to ____.

Ans. (368)



Sol.

$$A = \int_1^{25} 4\sqrt{x}\,dx - \frac{1}{2}\times4\times4 - \frac{1}{2}\times20\times20$$

$$A = \left[\frac{4x^{3/2}}{\frac{3}{2}}\right]_1^{25} - 8 - 200$$

$$A = \frac{8}{3}(125 - 1) - 208$$

$$A = \frac{368}{3} \Rightarrow 3A = 368$$

22. Let $A = \begin{bmatrix}\cos\theta & 0 & -\sin\theta\\ 0 & 1 & 0\\ \sin\theta & 0 & \cos\theta\end{bmatrix}$. If for some $\theta \in (0, \pi)$, $A^2 = A^T$, then the sum of the diagonal elements of the matrix $(A + I)^3 + (A - I)^3 - 6A$ is equal to ____.

Ans. (6)

Sol. $\because$ A is orthogonal matrix

$\therefore A^T = A^{-1}$

$\Rightarrow A^2 = A^{-1}$ $\quad (\because A^2 = A^T)$

$\Rightarrow A^3 = I$

let $B = (A + I)^3 + (A - I)^3 - 6A$

$= 2(A^3 + 3A) - 6A$

$= 2A^3$

$$B = 2I = \begin{bmatrix}2 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 2\end{bmatrix}$$

Now sum of diagonal elements = 2 + 2 + 2 = 6

23. Let $A = \{z \in C : |z - 2 - i| = 3\}$, $B = \{z \in C : \text{Re}(z - iz) = 2\}$ and $S = A \cap B$. Then $\sum_{z\in S}|z|^2$ is equal to ____.

Ans. (22)

Sol. Let $z = x + iy$

$A : |z - 2 - i| = 3$

$|(x - 2) + (y - 1)i| = 3$

$(x - 2)^2 + (y - 1)^2 = 9$ ……(1)

$B = \text{Re}(z - iz) = 2$

$\text{Re}((x + y) + i(y - x)) = 2$

$x + y = 2$ ……(2)

On solving (1) and (2) we get

$$x = \frac{3\pm\sqrt{17}}{2},\ y = \frac{1\mp\sqrt{17}}{2}$$

$$\sum_{z\in s}|z|^2 = \frac{1}{4}[2\times26 + 2\times18]$$

$$\Rightarrow \frac{88}{4} = 22$$

24. Let C be the circle $x^2 + (y-1)^2 = 2$, E_1 and E_2 be two ellipses whose centres lie at the origin and major axes lie on x-axis and y-axis respectively. Let the straight line $x + y = 3$ touch the curves C, E_1 and E_2 at $P(x_1, y_1)$, $Q(x_2, y_2)$ and $R(x_3, y_3)$ respectively. Given that P is the mid-point of the line segment QR and $PQ = \frac{2\sqrt{2}}{3}$, the value of $9(x_1y_1 + x_2y_2 + x_3y_3)$ is equal to _____.

Ans. **(46)**

Sol. Let $E_1 : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,\ (a > b)$

$E_2 : \frac{x^2}{c^2} + \frac{y^2}{d^2} = 1,\ (c < d)$

$C : x^2 + (y-1)^2 = 2$

Equation of tangent at $P(x_1,y_1)$

$xx_1 + y(y_1 - 1) = (y_1 + 1)$

comparing with $x + y = 3$ we get P(1,2)

$\because$ Now parametric equation of $x + y = 3$

$$\frac{(x-1)}{\left(\frac{-1}{\sqrt{2}}\right)} = \frac{(y-2)}{\left(\frac{1}{\sqrt{2}}\right)} = \pm\frac{2\sqrt{2}}{3} \qquad \left(\because PQ = \frac{2\sqrt{2}}{3}\right)$$

On solving we get $Q\left(\frac{5}{3}, \frac{4}{3}\right), R\left(\frac{1}{3}, \frac{8}{3}\right)$

So, $9(x_1y_1 + x_2y_2 + x_3y_3)$

$9\left(2 + \frac{5}{3} \times \frac{4}{3} + \frac{1}{3} \times \frac{8}{3}\right)$

$\Rightarrow 46$

25. Let m and n be the number of points at which the function $f(x) = \max\{x, x^3, x^5, \ldots, x^{21}\}$, $x \in \mathbb{R}$, is not differentiable and not continuous, respectively. Then m + n is equal to _____.

Ans. **(3)**

Sol. $$f(x) = \begin{cases} x, & x < -1 \\ x^{21}, & -1 \le x < 0 \\ x, & 0 \le x < 1 \\ x^{21}, & x \ge 1 \end{cases}$$

f(x) is continuous everywhere.

$\therefore n = 0$

$$f'(x) = \begin{cases} 1, & x < -1 \\ 21x^{20}, & -1 \le x < 0 \\ 1, & 0 < x < 1 \\ 21x^{20}, & x \ge 1 \end{cases}$$

$\therefore$ f(x) is non-differentiable at $x = -1, 0, 1$

$\therefore m = 3$

$m + n = 3$

SECTION-A

26. The mean free path and the average speed of oxygen molecules at 300 K and 1 atm are 3×10^{-7} m and 600 m/s, respectively. Find the frequency of its collisions.

(1) 2×10^{10}/s (2) 9×10^{5}/s

(3) 2×10^{9}/s (4) 5×10^{8}/s

Ans. **(3)**

Sol. Frequency = $\frac{1}{T} = \frac{V_{avg}}{\lambda}$

$$= \frac{600}{3 \times 157} = 2 \times 10^{9}\ \text{sec}^{-1}$$

27. A small mirror of mass m is suspended by a massless thread of length *l*. Then the small angle through which the thread will be deflected when a short pulse of laser of energy E falls normal on the mirror

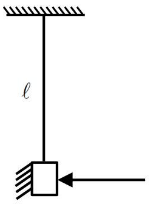
(c = speed of light in vacuum and g = acceleration due to gravity)

(1) $\theta = \frac{3E}{4mc\sqrt{gl}}$ (2) $\theta = \frac{E}{mc\sqrt{gl}}$

(3) $\theta = \frac{E}{2mc\sqrt{gl}}$ (4) $\theta = \frac{2E}{mc\sqrt{gl}}$

Ans. **(4)**

Sol.



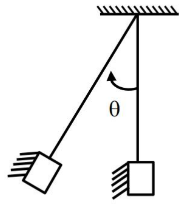
Force due to beam assuming complete reflection

$F = \frac{2P}{C} = \frac{2}{C}\frac{dE}{dt}$; P is power

So change in momentum of mirror.

$$m\,(V-0) = \int F dt = \frac{2}{C}\int dE = \frac{2E}{C}$$

Now using work energy theorem ....(1)



$W_g = \Delta k$

$$-mg\ell(1-\cos\theta) = 0 - \frac{1}{2}mv^2$$

$$g\ell\left(2\sin^2\frac{\theta}{2}\right) = \frac{v^2}{2}$$

as θ is small

$$g\ell 2\left(\frac{\theta}{2}\right)^2 = \frac{1}{2}\frac{4E^2}{m^2c^2} \quad \text{(from eq. (1))}$$

$$g\ell\theta^2 = \frac{4E^2}{m^2c^2}$$

$$\theta = \frac{2E}{mc\sqrt{g\ell}}$$

28. Two liquids A and B have θ_A and θ_B as contact angles in a capillary tube. If $K = \cos\theta_A/\cos\theta_B$. then identify the correct statement:

(1) K is negative, then liquid A and liquid B have convex meniscus.

(2) K is negative, then liquid A and liquid B have concave meniscus.

(3) K is negative, then liquid A has concave meniscus and liquid B has convex meniscus

(4) K is zero, then liquid A has convex meniscus and liquid B has concave meniscus.

Ans. **(3)**

Sol. $k = \frac{\cos\theta_A}{\cos\theta_B}$

It is negative when $\cos\theta_A$ & $\cos\theta_B$ are of opposite sign. so option (3)

29. Which of the following are correct expression for torque acting on a body?

A. $\vec{\tau} = \vec{r} \times \vec{L}$

B. $\vec{\tau} = \frac{d}{dt}(\vec{r} \times \vec{p})$

C. $\vec{\tau} = \vec{r} \times \frac{d\vec{p}}{dt}$

D. $\vec{\tau} = I\vec{\alpha}$

E. $\vec{\tau} = \vec{r} \times \vec{F}$

($\vec{r}$ = position vector; $\vec{p}$ = linear momentum;

$\vec{L}$ = angular momentum; $\vec{\alpha}$ = angular acceleration;

I = moment of inertia; $\vec{F}$ = force; t = time)

Choose the correct answer from the options given below :

(1) B, D and E Only (2) C and D Only

(3) B, C, D and E Only (4) A, B, D and E Only

Ans. **(3)**

Sol. Conceptual

30. In a Young's double slit experiment, the slits are separated by 0.2 mm. If the slits separation is increased to 0.4 mm, the percentage change of the fringe width is:

(1) 0% (2) 100%

(3) 50% (4) 25%

Ans. **(3)**

Sol. $\beta = \frac{D\lambda}{d} \propto \frac{1}{d}$

If d is doubled then β is half so 50% decrement.

31. An alternating current is represented by the equation,

$i = 100\sqrt{2}\sin(100\pi t)$ ampere. The RMS value of current and the frequency of the given alternating current are

(1) $100\sqrt{2}\,A, 100\,Hz$ (2) $\frac{100}{\sqrt{2}}A, 100\,Hz$

(3) 100 A, 50 Hz (4) $50\sqrt{2}\,A, 50\,Hz$

Ans. **(3)**

Sol. $i_r = \frac{i_0}{\sqrt{2}} = 100A$

$f = \frac{w}{2\pi} = \frac{100\pi}{2\pi} = 50Hz$

32. Consider the sound wave travelling in ideal gases of He, CH_4, and CO_2. All the gases have the same ratio $\frac{P}{\rho}$, where P is the pressure and ρ is the density. The ratio of the speed of sound through the gases $v_{He} : v_{CH_4} : v_{CO_2}$ is given by

(1) $\sqrt{\frac{7}{5}} : \sqrt{\frac{5}{3}} : \sqrt{\frac{4}{3}}$ (2) $\sqrt{\frac{5}{3}} : \sqrt{\frac{4}{3}} : \sqrt{\frac{7}{5}}$

(3) $\sqrt{\frac{5}{3}} : \sqrt{\frac{4}{3}} : \sqrt{\frac{4}{3}}$ (4) $\sqrt{\frac{4}{3}} : \sqrt{\frac{5}{3}} : \sqrt{\frac{7}{5}}$

Ans. **(3)**

Sol. $v_{sound} = \sqrt{\frac{\gamma p}{\rho}}$

$\gamma = 1 + \frac{2}{f}$

$\gamma_{He} = \frac{5}{3}$;

$\gamma_{CH_4} = \gamma_{CO_2} \approx 1.33 = \frac{4}{3}$ (Experimental data)

33. In an electromagnetic system, the quantity representing the ratio of electric flux and magnetic flux has dimension of $M^P L^Q T^R A^S$, where value of 'Q' and 'R' are

(1) (3, –5) (2) (–2, 2)

(3) (–2, 1) (4) (1, –1)

Ans. **(4)**

Sol. $\frac{\phi_E}{\phi_M} = \frac{EA}{BA} = \frac{E}{B}$

$B = \frac{M\ell T^{-2}}{ATLT^{-1}}$

So $\left[\frac{E}{B}\right] = \frac{ML^{-3}A^{-1}}{MT^{-2}A^{-1}} = LT^{-1}$

Or

$E = c.B$ (c = Speed of light)

$$\left[\frac{E}{B}\right] = LT^{-1}$$

34. When an object is placed 40 cm away from a spherical mirror an image of magnification $\frac{1}{2}$ is produced. To obtain an image with magnification of $\frac{1}{3}$, the object is to be moved :

(1) 40 cm away from the mirror.

(2) 80 cm away from the mirror.

(3) 20 cm towards the mirror.

(4) 20 cm away from the mirror.

Ans. (1)

Sol. $m = \frac{1}{2} = \frac{f}{f-u}$

$$\frac{1}{2} = \frac{f}{f-(-40)}$$

$f + 40 = 2f \Rightarrow f = 40$ cm

now $m = \frac{1}{3} = \frac{40}{40-u}$

$40 - u = 120 \Rightarrow u = -80$

35. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A: In photoelectric effect, on increasing the intensity of incident light the stopping potential increases.

Reason R : Increase in intensity of light increases the rate of photoelectrons emitted, provided the frequency of incident light is greater than threshold frequency.

In the light of the above statements, choose the **correct** answer from the options given below

(1) Both **A** and **R** are true but **R** is **NOT** the correct explanation of **A**

(2) **A** is false but **R** is true

(3) **A** is true but **R** is false

(4) Both **A** and **R** are true and **R** is the correct explanation of **A**

Ans. (2)

Sol. $V_S = \frac{h\nu - \phi}{e}$

so stopping potential doesn't depend on Intensity

$I = \frac{\eta h\nu}{A}$

On increasing intensity no. of photons per sec. n increases so the no. of electrons.

36. If $\vec{L}$ and $\vec{P}$ represent the angular momentum and linear momentum respectively of a particle of mass 'm' having position vector $\vec{r} = a(\hat{i}\cos\omega t + \hat{j}\sin\omega t)$. The direction of force is

(1) Opposite to the direction of $\vec{r}$

(2) Opposite to the direction of $\vec{L}$

(3) Opposite to the direction of $\vec{P}$

(4) Opposite to the direction of $\vec{L} \times \vec{P}$

Ans. (1)

Sol. $\vec{a} = -\omega^2 \vec{r}$

$\therefore$ $\vec{F}$ opposite to $\vec{r}$ -

37. A body of mass m is suspended by two strings making angles θ_1 and θ_2 with the horizontal ceiling with tensions T_1 and T_2 simultaneously. T_1 and T_2 are related by $T_1 = \sqrt{3}T_2$. the angles θ_1 and θ_2 are

(1) $\theta_1 = 30°\ \theta_2 = 60°$ with $T_2 = \frac{3mg}{4}$

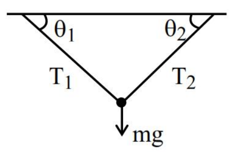
(2) $\theta_1 = 60°\ \theta_2 = 30°$ with $T_2 = \frac{mg}{2}$

(3) $\theta_1 = 45°\ \theta_2 = 45°$ with $T_2 = \frac{3mg}{4}$

(4) $\theta_1 = 30°\ \theta_2 = 60°$ with $T_2 = \frac{4mg}{5}$

Ans. (2)

Sol.



$T_1 \sin\theta_1 + T_2 \sin\theta_2 = mg$ & $T_1 = \sqrt{3}\,T_2$

$\Rightarrow T_2\left[\sqrt{3}\sin\theta_1 + \sin\theta_2\right] = mg$

for $\theta_1 = 60°$ & $\theta_2 = 30°$

$T_2 = \frac{mg}{2}$

38. Current passing through a wire as function of time is given as I(t) = 0.02 t + 0.01 A. The charge that will flow through the wire from t = 1s to t = 2s is :

(1) 0.06 C (2) 0.02 C

(3) 0.07 C (4) 0.04 C

Ans. (4)

Sol. $q = \int i\,dt$

$\int_0^2 (0.02t + 0.01)dt$

$q = \left[0.02\frac{t^2}{2} + 0.01t\right]_1^2$

= 0.01 (3) + 0.01 (1)

= 0.04 C

39. Given below are two statements : one is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A : The kinetic energy needed to project a body of mass m from earth surface to infinity is $\frac{1}{2}$ mgR, where R is the radius of earth.

Reason R : The maximum potential energy of a body is zero when it is projected to infinity from earth surface.

In the light of the above statements, choose the **correct** answer from the option given below

(1) **A** False but **R** is true

(2) Both **A** and **R** are true and **R** is the correct explanation of **A**

(3) **A** is true but **R** is false

(4) Both **A** and **R** are true but **R** is **NOT** the correct explanation of **A**

Ans. (1)

Sol. $KE = \frac{1}{2}m\left(\frac{2Gm}{R}\right) = mgR$

Assertion wrong

at ∞ U = 0

∴ Reason correct.

40. The Boolean expression $Y = A\bar{B}C + \overline{A}\overline{C}$ can be realised with which of the following gate configurations.

A. One 3-input AND gate, 3 NOT gates and one 2-input OR gate, One 2-input AND gate,

B. One 3-input AND gate, 1 NOT gate, One 2-input NOR gate and one 2-input OR gate

C. 3-input OR gate, 3 NOT gates and one 2-input AND gate

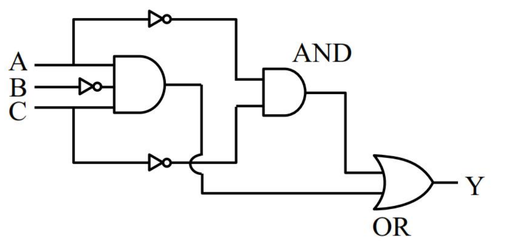
Choose the **correct** answer from the options given below

(1) B, C Only (2) A,B Only

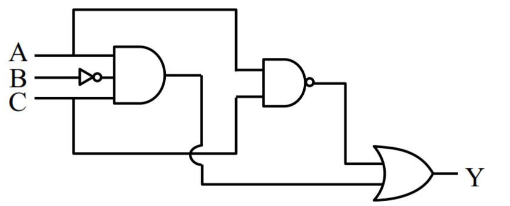
(3) A, B, C Only (4) A, C Only

Ans. (2)

Sol. (A)



(B)



$\because\ \overline{A}\cdot\overline{C} + \overline{A+C} \equiv$ NOR gate

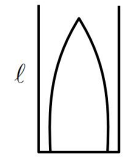
41. In an experiment with a closed organ pipe, it is filled with water by $\left(\frac{1}{5}\right)$th of its volume. The frequency of the fundamental note will change by

(1) 25% (2) 20%

(3) –20% (4) –25%

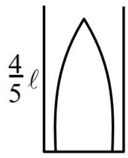
Ans. (1)

Sol.



$\lambda_1 = 4\ell$

$f_1 = \dfrac{v}{4\ell}$



$\lambda_2 = \dfrac{16\ell}{5}$

$f_2 = \dfrac{5V}{16\ell}$

$$\frac{\Delta f}{f} = \frac{\frac{V}{\ell}\left(\frac{1}{16}\right)}{\frac{V}{4\ell}} \times 100 = 25\%$$

42. Two simple pendulums having lengths l_1 and l_2 with negligible string mass undergo angular displacements θ_1 and θ_2, from their mean positions, respectively. If the angular accelerations of both pendulums are same, then which expression is correct ?

(1) $\theta_1 l_2^2 = \theta_2 l_1^2$ (2) $\theta_1 l_1 = \theta_2 l_2$

(3) $\theta_1 l_1^2 = \theta_2 l_2^2$ (4) $\theta_1 l_2 = \theta_2 l_1$

Ans. **(4)**

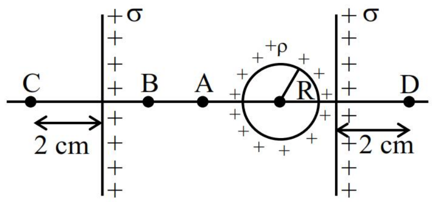
Sol. $\omega = \sqrt{\dfrac{g}{\ell}}$

$\alpha = -\omega^2\theta$

$\therefore \dfrac{g}{\ell_1}\theta_1 = \dfrac{g}{\ell_2}\theta_2$

$\Rightarrow \theta_1\ell_2 = \theta_2\ell_2$

43. Two infinite identical charged sheets and a charged spherical body of charge density 'ρ' are arranged as shown in figure. Then the correct relation between the electrical fields at A, B, C and D points is :



(1) $\vec{E}_A = \vec{E}_B; \vec{E}_C = \vec{E}_D$ (2) $\vec{E}_A > \vec{E}_B; \vec{E}_C = \vec{E}_D$

(3) $\vec{E}_C \neq \vec{E}_D; \vec{E}_A > \vec{E}_B$ (4) $|\vec{E}_A| = |\vec{E}_B|; \vec{E}_C > \vec{E}_D$

Ans. **(3)**

Sol. Conceptual

$E_C \neq E_D$

$E_A > E_B$

44. Two small spherical balls of mass 10g each with charges –2μC and 2μC, are attached to two ends of very light rigid rod of length 20 cm. The arrangement is now placed near an infinite non-conducting charge sheet with uniform charge density of $100\mu C/m^2$ such that length of rod makes an angle of 30° with electric field generated by charge sheet. Net torque acting on the rod is:

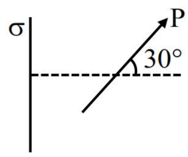
(Take ε_o : 8.85×10^{-12} C^2/Nm^2)

(1) 112 Nm (2) 1.12 Nm

(3) 2.24 Nm (4) 11.2 Nm

Ans. **(2)**

Sol.



$E = \dfrac{\sigma}{2\varepsilon_0}$

$\tau = PE\sin\theta$

$$= \left[\left(2\times10^{-6}\right)\left(\frac{2}{10}\right)\right]\left[\frac{100\times10^{-6}}{2\times8.85\times10^{-12}}\right]\left(\frac{1}{2}\right)$$

$$= \frac{10}{8.85} = 1.12\,\text{Nm}$$

45. Considering the Bohr model of hydrogen like atoms, the ratio of the ratio of the radius 5^{th} orbit of the electron in Li^{2+} and He^{+} is

(1) $\frac{3}{2}$ (2) $\frac{4}{9}$

(3) $\frac{9}{4}$ (4) $\frac{2}{3}$

Ans. (4)

Sol. $r = r \cdot \frac{n^2}{2}$

for Li^{2+}

$r_5 = r \cdot \frac{25}{3}$

for He^{+}

$r_5 = r \cdot \frac{25}{2}$

$\therefore \frac{r_{Li^{2+}}}{r_{He^{+}}} = \frac{2}{3}$

SECTION-B

46. A circular ring and a solid sphere having same radius roll down on an inclined plane from rest without slipping. The ratio of their velocities when reached at the bottom of the plane is $\sqrt{\frac{x}{5}}$ where x = _____.

Ans. (4)

Sol. Applying Mechanical Energy conservation :

$k_i + U_i = k_f + U_f$

$\Rightarrow 0 + Mgh = \frac{1}{2}mv^2\left(1+\frac{k^2}{R^2}\right)+0$

$\Rightarrow V = \sqrt{\frac{2gh}{1+\frac{k^2}{R^2}}}$

So Ratio of velocities

$\frac{V_{Ring}}{V_{solids\ sphere}} = \sqrt{\frac{1+\frac{2}{5}}{1+1}} = \sqrt{\frac{7}{10}}$

x = 3.5 Rounding off x = 4

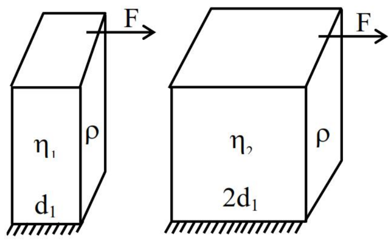
47. Two slabs with square cross section of different materials (1, 2) with equal sides (l) and thickness d_1 and d_2 such that $d_2 = 2d_1$ and $l > d_2$. Considering lower edges of these slabs are fixed to the floor, we apply equal shearing force on the narrow faces. The angle of deformation is $\theta_2 = 2\theta_1$. If the shear moduli of material 1 is 4×10^9 N/m^2, then shear moduli of material 2 is x $\times 10^9$ N/m^2, where value of x is ____.

Ans. (1)

Sol. Deformation angle

$2\theta_1 = \theta_2$

$\Rightarrow 2\frac{\sigma_1}{\eta_1} = \frac{\sigma_2}{\eta_2}$



$\Rightarrow 2\left(\frac{F}{\ell d_1 \eta_1}\right) = \frac{F}{\ell d_2 \eta_2}$

$\Rightarrow \eta_2 = \frac{\eta_1}{4} = 1\times10^9 \quad \Rightarrow x = 1$

48. Distance between object and its image (magnified by $-\frac{1}{3}$) is 30 cm. The focal length of the mirror used is $\left(\frac{x}{4}\right)$ cm,

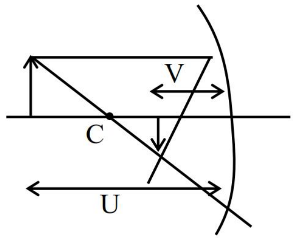
where magnitude of value of x is _____.

Ans. (45)

Sol. $M = -\frac{1}{3}$

$-\frac{-V}{-U} = \frac{-1}{3} \Rightarrow V = \frac{U}{3}$

Distance b/w object and image :



U – V = 30

$U - \frac{\mu}{3} = 30$

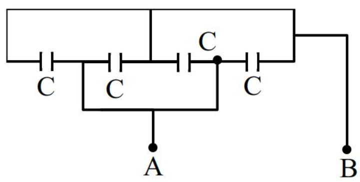
$\Rightarrow U = 45 \quad V = 15$

$\frac{1}{f} = \frac{1}{V} + \frac{1}{U} = -\frac{1}{15} - \frac{1}{45}$

$\Rightarrow F = \frac{45}{4}$

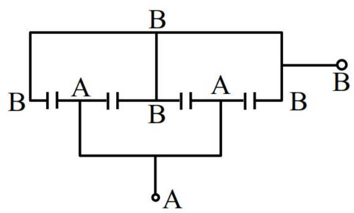
x = 45

49. Four capacitor each of capacitance 16μF are connected as shown in the figure. The capacitance between points A and B is : _____ (in μF).

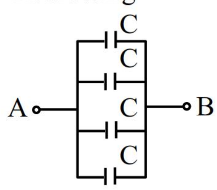


Ans. (64)

Sol.

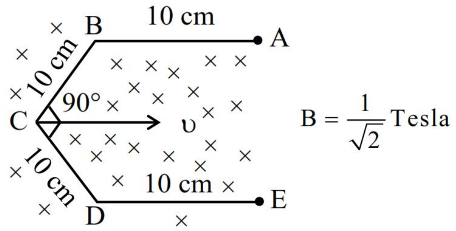


Redrawing



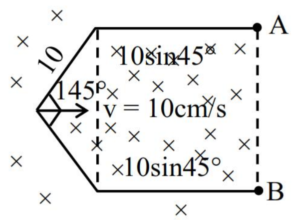
$C_{eq} = 4C = 64$

50. Conductor wire ABCDE with each arm 10 cm in length is placed in magnetic field of $\frac{1}{\sqrt{2}}$ Tesla, perpendicular to its plane. When conductor is pulled towards right with constant velocity of 10 cm/s, induced emf between points A and E is ________ mV.



Ans. (10)

Sol.



As field is uniform we can replace the bent wire with straight wire from A to B.

So EMF :

$\varepsilon = Bv\ell_{AB}$

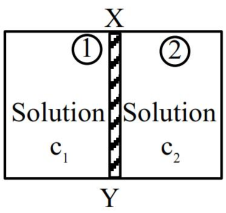
$= \frac{1}{\sqrt{2}} \times \frac{10cm}{5} \times 2(10\sin 45°)cm$

ε = 10 mV

SECTION-A

51. *XY* is the membrane / partition between two chambers 1 and 2 containing sugar solutions of concentration c_1 and c_2 ($c_1 > c_2$) mol L^{-1}. For the reverse osmosis to take place identify the correct condition

(Here p_1 and p_2 are pressures applied on chamber 1 and 2)



(A) Membrane/Partition ; Cellophane, $p_1 > \pi$

(B) Membrane/Partition ; Porous. $p_2 > \pi$

(C) Membrane/Partition ; Parchment paper, $p_1 > \pi$

(D) Membrane/Partition : Cellophane, $p_2 > \pi$

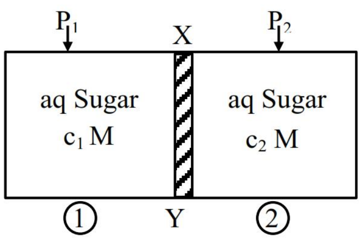
Choose the **correct** answer from the option given below :

(1) B and D only (2) A and D only

(3) A and C only (4) C only

Ans. (3)

Sol.



Given $C_1 > C_2$

Normal osmosis occurs from (2) to (1)

For reverse osmosis from (1) to (2)

Pressure : $P_1 > \pi$

$\therefore$ Answer [A & C] only

52. Let us consider a reversible reaction at temperature, T.

In this reaction, both ΔH and ΔS were observed to have positive values. If the equilibrium temperature is Te, then the reaction becomes spontaneous at :

(1) T = Te (2) Te > T

(3) T > Te (4) Te = 5T

Ans. (3)

Sol. For reaction to be spontaneous according to 2^{nd} law:

$\Delta G < 0$

$\Rightarrow \Delta H - T\Delta S < 0$

$$\Rightarrow T > \left(\frac{\Delta H}{\Delta S}\right) = T_e$$

$\Rightarrow T > T_e$

53. Which of the following molecules(s) show/s paramagnetic behavior ?

(A) O_2 (B) N_2 (C) F_2 (D) S_2 (E) Cl_2

Choose the **correct** answer from the options given below :

(1) B only (2) A & C only

(3) A & E only (4) A & D only

Ans. (4)

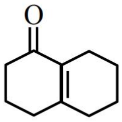
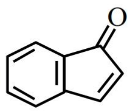
Sol.

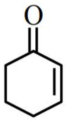
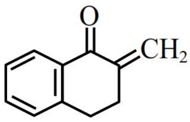
		No. of unpaired e^-
(A)	O_2	2
(B)	N_2	0
(C)	F_2	0
(D)	S_2	2
(E)	Cl_2	0

If species contain unpaired electron than it is paramagnetic.

So A & D are paramagnetic.

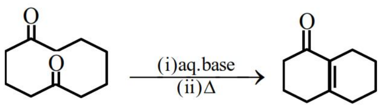
54. Aldol condensation is a popular and classical method to prepare α,β-unsaturated carbonyl compounds. This reaction can be both intermolecular and intramolecular. Predict which one of the following is not a product of intramolecular aldol condensation ?

(1)  (2) 

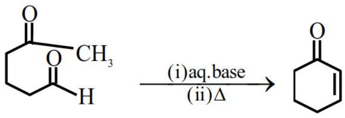
(3)  (4) 

Ans. (4)

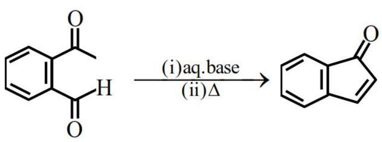
Sol.

(1) 

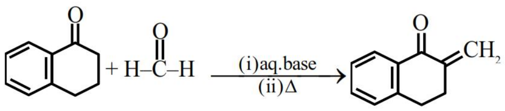
(Intramolecular aldol)

(2) 

(Intramolecular aldol)

(3) 

(Intramolecular aldol)

(4) 

(Intermolecular aldol)

55. One mole of an ideal gas expands isothermally and reversibly from 10 dm^3 to 20 dm^3 at 300 K. ΔU, q and work done in the process respectively are :

Given : R = 8.3 JK^{-1} and mol^{-1}

In 10 = 2.3

log 2 = 0.30

log 3 = 0.48

(1) 0, 21.84 kJ, –1.26 kJ (2) 0, –17.18 kJ, 1.718 J

(3) 0, 21.84 kJ, 21,84 kJ (4) 0,178 kJ, – 1.718 kJ

Ans. (4)

Sol. $(10L, 300K) \xrightarrow{n=1} (20L, 300K)$

$-q = w = -nRT \ln\frac{V_2}{V_1}$

$= -8.3 \times 300 \times \ln\left(\frac{20}{10}\right)$

$= -1.718$ kJ

$\Rightarrow$ q = 1.718 kJ

w = –1.718 kJ

ΔU = 0 ($\because$ ΔT = 0)

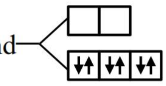
56. Which one of the following complexes will have $\Delta_0 = 0$ and μ = 5.96 B.M.?

(1) $[Fe(CN)_6]^{4-}$ (2) $[CO(NH_3)_6]^{3+}$

(3) $[FeF_6]^{4-}$ (4) $[Mn(SCN)_6]^{4-}$

Ans. (4)

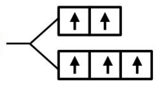
Sol. (1) $[Co(NH_3)_6]^{3+}$ $Co^{3+} \Rightarrow 3d^6 4s^0$

NH_3 is strong field ligand  μ = 0

$= [-0.4 \times 6 + 0.6 \times (0)]\Delta_0 = -2.4\ \Delta_0$

(2) $[Mn(SCN)_6]^{4-}$

$Mn^{2+} \Rightarrow 3d^5\ 4s^0$

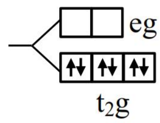
$\overset{\ominus}{S}CN$ is weak field 

$\mu = \sqrt{35}$ B.M. = 5.96 B.M.

CFSE = $(-0.4 \times 3 + 0.6 \times 2)\Delta_0$

So $\Delta_0 = 0$

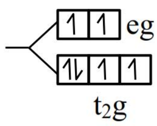
(3) $[Fe(CN)_6]^{-4}$ $Fe^{2+} \Rightarrow 3d^6 4s^0$

$\overset{\ominus}{C}N$ is SFL 

μ = 0

CFSE = $-2.4\Delta_0$

(4) $[FeF_6]^{4-}$ $Fe^{2+} \Rightarrow 3d^6 4s^0$



$\mu = \sqrt{24}$ B.M. = 4.89 B.M.

CFSE = $(-0.4 \times 4 + 0.6 \times 2)\Delta_0 = -1.2\Delta_0$

57. For $A_2 + B_2 \rightleftharpoons 2AB$

E_a for forward and backward reaction are 180 and 200 kJ mol^{-1} respectively

If catalyst lowers E_a for both reaction by 100 kJ mol^{-1}.

Which of the following statement is correct?

(1) Catalyst does not alter the Gibbs energy change of a reaction.

(2) Catalyst can cause non-spontaneous reactions to occur.

(3) The enthalpy change for the reaction is +20 kJ mol^{-1}.

(4) The enthalpy change for the catalysed reaction is different from that of uncatalysed reaction.

Ans. **(1)**

Sol. $A_2 \quad + \quad B_2 \quad \rightleftharpoons \quad 2AB$

$E_f = 180$ kJ mol^{-1}

$E_b = 200$ kJ mol^{-1}

$\Delta H = E_f - E_b = -20$ kJ mol^{-1}

In presence of catalyst :

$E_f = 180 - 100 = 80$ kJ mol^{-1}

$E_b = 200 - 100 = 100$ kJ mol^{-1}

Catalyst does not change ΔH or ΔG of a reaction.

58. Rate law for a reaction between A and B is given by

$R = k[A]^n[B]^m$

If concentration of A is doubled and concentration of B is halved from their initial value, the ratio of new rate of reaction to the initial rate of reaction $\left(\frac{r_2}{r_1}\right)$ is

(1) $2^{(n-m)}$ (2) (n–m)

(3) (m + n) (4) $\frac{1}{2^{m+n}}$

Ans. **(1)**

Sol. $r_1 = k[A]^n[B]^m$

Now A is doubled & B is halved in concentration

$$\Rightarrow r_2 = k2^n[A]^n.\frac{[B]^m}{2^m}$$

Now $\frac{r_2}{r_1} = 2^{(n-m)}$

59. Number of stereoisomers possible for the complexes, $[CrCl_3(py)_3]$ and $[CrCl_2(ox)_2]^{3-}$ are respectively

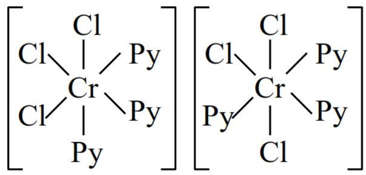
(py = pyridine, ox = oxalate)

(1) 3 & 3 (2) 2 & 2

(3) 2 &3 (4) 1 & 2

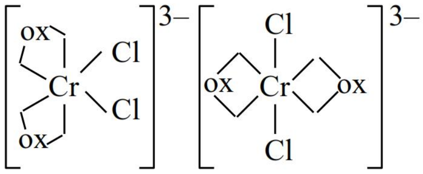
Ans. **(3)**

Sol.

$[CrCl_3(Py)_3]$ 

Facial Meridional

So total stereo isomer = 2

$[CrCl_3(ox)_2]^{3-}$ 

cis isomer trans isomer

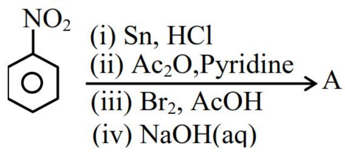
(Optically active)

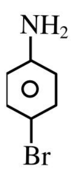
Geometrical isomer = 2 (1 cis +1 trans)

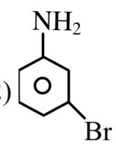
Optical isomer = 3 (2 optically active + 1 optically inactive)

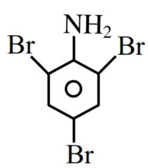
Stereoisomer = 3

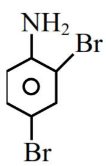
60. The major product (A) formed in the following reaction sequence is



(1) 

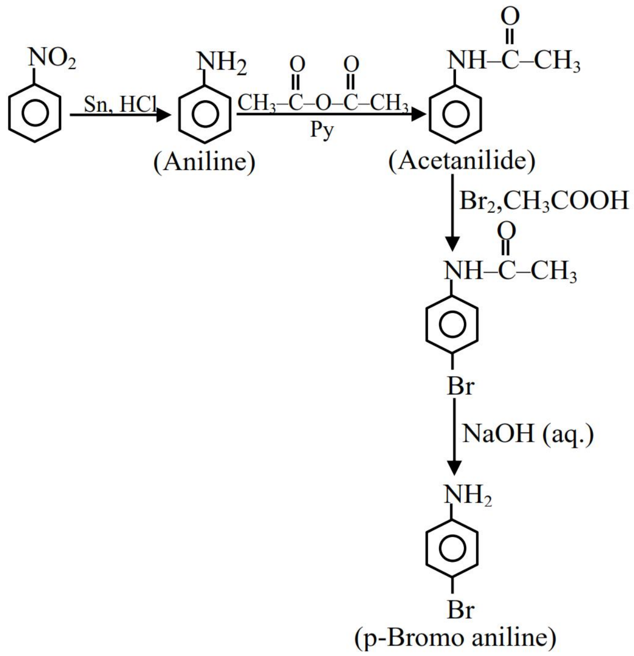
(2) 

(3) 

(4) 

Ans. **(1)**

Sol.



61. On charging the lead storage battery, the oxidation state of lead changes from x_1 to y_1 at the anode and from x_2 to y_2 at the cathode. The values of x_1,y_1,x_2,y_2 are respectively :

(1) +4,+2,0,+2 (2) +2,0,+2,+4

(3) 0,+2,+4,+2 (4) +2,0,0,+4

Ans. **(2)**

Sol. For charging of lead storage battery cell reaction is

$2PbSO_4(s)+2H_2O(l)\rightarrow Pb(s)+PbO_2(s)+2H_2SO_4(aq)$

At anode $PbSO_4$ reduced back to Pb and at cathode $PbSO_4$ oxidised back to PbO_2.

$\because$ $x_1 = +2$, $y_1 = 0$

$x_2 = +2$, $y_2 = 4$

62. Given below are two statements :

Statement I : Nitrogen forms oxides with +1 to +5 oxidation states due to the formation of $p\pi$ - $p\pi$ bond with oxygen .

Statement II : Nitrogen does not form halides with +5 oxidation state due to the absence of d-orbital in it.

In the light of given statements, choose the ***correct*** answer from the options given below.

(1) Statement I is true but Statement II is false

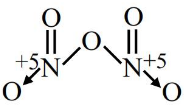
(2) Both Statement I and Statement II are false

(3) Statement I is false but Statement II is true

(4) Both Statement I and Statement II are true

Ans. **(4)**

Sol. In oxide of nitrogen it can achieve +5 oxidation state because it can form $p\pi-p\pi$ bond with oxygen e.g. N_2O_5

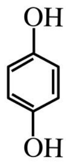


Nitrogen cannot form halide in +5 oxidation state because it does not contain d-orbital.

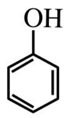
e.g. NX_5 does not exist

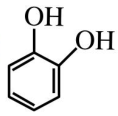
X = halide

63. Benzene is treated with oleum to produce compound (X) which when further heated with molten sodium hydroxide followed by acidification produces compound (Y).The compound Y is treated with zinc metal to produce compound (Z). Identify the structure of compound (Z) from the following option.

(1) 

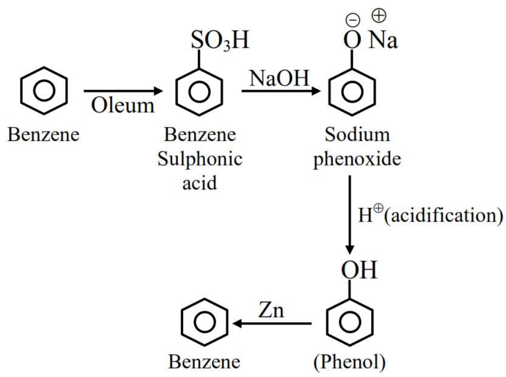
(2) 

(3) 

(4) 

Ans. **(2)**

Sol.



64. Identify the pair of reactants that upon reaction, with elimination of HCl will give rise to the dipeptide Gly-Ala.

(1) $NH_2–CH_2–COCl$ and $NH_2–CH(CH_3)–COOH$

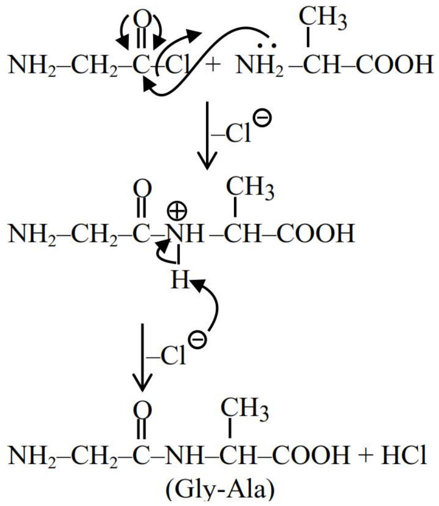
(2) $NH_2–CH_2–COCl$ and $\overset{\oplus}{N}H_3–CH(CH_3)–COCl$

(3) $NH_2–CH_2–COOH$ and $NH_2–CH(CH_3)–COCl$

(4) $NH_2–CH_2–COOH$ and $NH_2–CH(CH_3)–COOH$

Ans. **(1)**

Sol.



65. Given below are the pairs of group 13 elements showing their relation in terms of atomic radius.

(B<Al), (Al<Ga), (Ga<In) and (In<Tl)

Identify the elements present in the incorrect pair and in that pair find out the element (X) that has higher ionic radius (M^{3+}) that the other one. The atomic number of the element (X) is

(1) 31 (2) 49

(3) 13 (4) 81

Ans. **(1)**

Sol. Size order

Al > Ga

$Al^{3+} < Ga^{3+}$

Atomic number of Ga is 31

66. An organic compound (X) with molecular formula C_3H_6O is not readily oxidised. On reduction it gives (C_3H_8O(Y) which reacts with HBr to give a bromide (Z) which is converted to Grignard reagent. This Grinard reagent on reaction with (X) followed by hydrolysis give 2,3-dimethylbutan-2-ol. Compounds (X), (Y) and (Z) respectively are :

(1) CH_3COCH_3, $CH_3CH_2CH_2OH$, $CH_3CH(Br)\ CH_3$

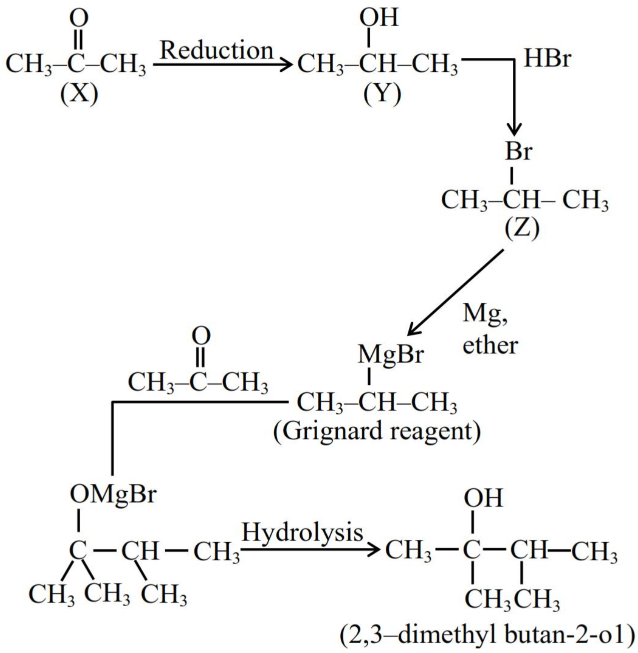
(2) CH_3COCH_3, $CH_3CH(OH)CH_3$, $CH_3CH(Br)CH_3$

(3) CH_3CH_2CHO, $CH_3CH_2CH_2OH$, $CH_3CH_2CH_2Br$

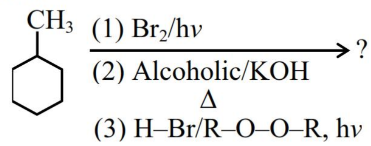
(4) CH_3CH_2CHO, $CH_3CH = CH_2$,$CH_3CH(Br)\ CH_3$

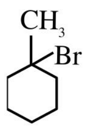
Ans. (2)

Sol.

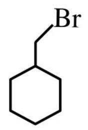


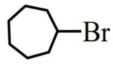
67. Predict the major product of the following reaction sequence :-



(1) 

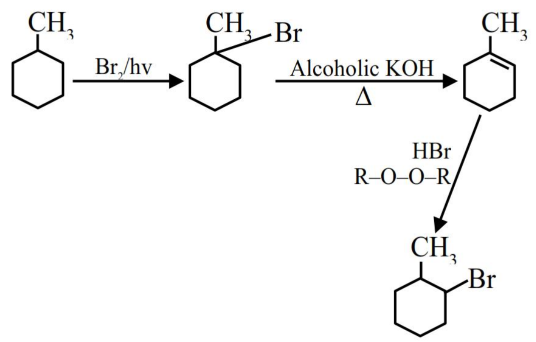
(2) 

(3) 

(4) 

Ans. **(2)**

Sol.



68. Given below are two statements.

Statement I : The dipole moment of $\overset{4}{C}H_3-\overset{3}{C}H=\overset{2}{C}H-\overset{1}{C}H=O$ is greater than $\overset{4}{C}H_3-\overset{3}{C}H_2-\overset{2}{C}H_2-\overset{1}{C}H=O$

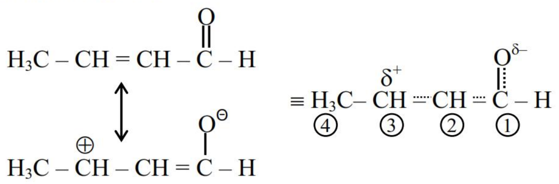
Statement II : $C_1–C_2$ bond length of $\underset{4}{C}H_3-\underset{3}{C}H=\underset{2}{C}H-\underset{1}{C}H=O$ is greater than $C_1–C_2$ bond length of $\underset{4}{C}H_3-\underset{3}{C}H_2-\underset{2}{C}H_2-\underset{1}{C}H=O$

In the light of the above statements, choose the ***correct*** answer from the options given below:

(1) Statement I is false but Statement II is true
(2) Both Statement I and Statement II are false
(3) Statement I is true but Statement II is false
(4) Both Statement I and Statement II are true

Ans. (3)

Sol. Statement-I :

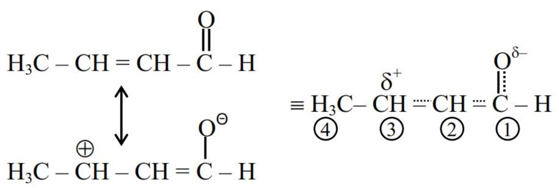


$\mu = q \times d$

More charges and more distance between charges than other compound so more dipole moment.

Statement-I is true.

Statement-II :



$C_1 - C_2$ bond has partial double bond character that means lesser bond length than $C_1 - C_2$ bond of other compound.

Statement-II is false.

69. Pair of transition metal ions having the same number of unpaired electrons is :

(1) V^{2+}, Co^{2+} (2) Ti^{2+}, Co^{2+}
(2) Fe^{3+}, Cr^{2+} (2) Ti^{3+}, Mn^{2+}

Ans. (1)

Sol.

			Configuration	No. of unpaired e^-
(1)	V^{3+}	⇒	$[Ar]3d^3 4s^0$	3
	Co^{2+}	⇒	$[Ar]3d^7 4s^0$	3
(2)	Ti^{2+}	⇒	$[Ar]3d^2 4s^0$	2
	Co^{2+}	⇒	$[Ar]3d^7 4s^0$	3
(3)	Fe^{3+}	⇒	$[Ar]3d^5 4s^0$	5
	Cr^{2+}	⇒	$[Ar]3d^4 4s^0$	4
(4)	Ti^{3+}	⇒	$[Ar]3d^1 4s^0$	1
	Mn^{2+}	⇒	$[Ar]3d^5 4s^0$	5

So V^{2+} & Co^{2+} same number of unpaired electron.

70. Which one of the following about an electron occupying the 1s orbital in a hydrogen atom is incorrect ? (Bohr's radius is represented by a_0)

(1) The probability density of finding the electron is maximum at the nucleus

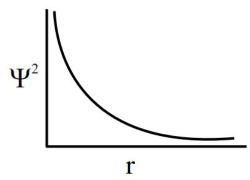
(2) The electron can be found at a distance $2a_0$ from the nucleus

(3) The 1s orbital is spherically symmetrical

(4) The total energy of the electron is maximum when it is at a distance a_0 from the nucleus

Ans. (4)

Sol.



1. Ψ^2 = Probability density is maximum at nucleus.
2. Electron can exist upto infinity from nucleus.
3. True
4. Energy of electron is maximum at infinite distance from nucleus.

SECTION-B

71. In Dumas' method for estimation of nitrogen 1g of an organic compound gave 150 mL of nitrogen collected at 300K temperature and 900 mm Hg pressure. The percentage composition of nitrogen in the compound is _____ % (nearest integer).

(Aqueous tension at 300 K = 15mm Hg)

Ans. (20)

Sol. Partial pressure of N_2 = (900 – 15) = 885 mm Hg

$$\text{Mole of } N_2 = \frac{\left(\frac{885}{760}\times 0.15\right)}{(0.0821\times 300)} = 0.0071 \text{ moles}$$

% of nitrogen in organic compound

$$= \frac{(0.0071\times 28)}{1}\times 10$$

= 19.85%

72. $KMnO_4$ acts as an oxidising agent in acidic medium. 'X' is the difference between the oxidation states of Mn in reactant and product. 'Y' is the number of 'd' electrons present in the brown red precipitate formed at the end of the acetate ion test with neutral ferric chloride. The value of X + Y is _____.

Ans. (10)

Sol. $$\underset{(O.A)}{\overset{+7}{KMnO_4}} \xrightarrow{\text{Acidic medium}} Mn^{2+}$$

X is difference in oxidation state.

7 – 2 = 5

So X = 5

$$6CH_3COO^{\ominus} + Fe^{3+} + H_2O \rightarrow [Fe_3(OH_2)(CH_3COO)_6]^{\oplus} + 2H^{\oplus}$$

$$[Fe_3(OH)_2(CH_3COO)_6]^{\oplus} + 4H_2O \rightarrow \underset{\text{Brown red ppt}}{[Fe(OH)_2(CH_3COO]} + CH_3COOH + H^{\oplus}$$

$Fe^{3+} \Rightarrow 3d^5\,4s^0$ contains 5 d electrons

So Y = 5

X + Y = 5 + 5 = 10

73. Fortification of food with iron is done using $FeSO_4.7H_2O$. The mass in grams of the $FeSO_4.7H_2O$ required to achieve 12 ppm of iron in 150 kg of wheat is _____(Nearest integer)

[**Given :** Molar mass of Fe, S and O respectively are 56, 32 and 16 g mol^{-1}]

Ans. (9)

Sol. Let mass of iron = w g

$$\Rightarrow \frac{w}{150\times 10^3} \times 10^6 = 12$$

$\Rightarrow w = 150 \times 12 \times 10^{-3} = 1.8$ gm

Let mass of $FeSO_4{\cdot}7H_2O = w_1$ gm

$$\Rightarrow \text{Moles of Fe} = \frac{1.8}{56} = \left(\frac{w_1}{56+96+7\times 18}\right)$$

$\Rightarrow w_1 = 8.935$ gm

74. The pH of a 0.01 M weak acid HX ($K_a = 4 \times 10^{-10}$) is found to be 5. Now the acid solution is diluted with excess of water so that the pH of the solution changes to 6. The new concentration of the diluted weak acid is given as x × 10^{-4} M. The value of x is ________(nearest integer)

Ans. (25)

Sol. $HX_{(aq)} \rightleftharpoons H^+_{(aq)} + X^-_{(aq)}$ $\quad K_a = 4 \times 10^{-10}$

$0.01(1-\alpha)$ $\quad 0.01\alpha$ $\quad 0.01\alpha$ $\quad$ Not justified

$\Rightarrow 0.01\alpha = 10^{-5} \Rightarrow \alpha = 10^{-3}$

$K_a = 0.01\alpha^2 = 10^{-8}$

On dilution let final concentration of HX = c M

$Hx_{(aq)} \rightleftharpoons H^+_{(aq)} + X^-_{(aq)}$

$C(1-\alpha)$ $\quad C\alpha$ $\quad C\alpha$

$\Rightarrow C\alpha = 10^{-6}$ ...(1)

$$\frac{C\alpha^2}{1-\alpha} = K_a = 10^{-8} \quad ...(2)$$

$$\Rightarrow \frac{10^{-6}\alpha}{1-\alpha} = 10^{-8}$$

Data given is inconsistent & contradictory. This should be bonus.

75. The total number of hydrogen bonds of a DNA-double Helix strand whose one strand has the following sequence of bases is ______.

5' – G – G-C-A-A-A-T-C-G-G-C-T-A-3'

Ans. **(33)**

Sol. Two nucleic acid chains are wound about each other and held together by H bonds between pair of bases.

Adenine from two hydrogen bonds with thymine and Guanine form three hydrogen bond with cytosine.

5' G–G–C–A–A–A–T–C–G–G–C–T–A–3'

In given DNA strand total seven guanine and cytosine bases which form total 21 H-bonds and six adenine and thymine base which will form total 12 H-bonds with other DNA strand.

Total no. of H bonds = $7 \times 3 + 6 \times 2 = 33$

Ans. 33