Class- X Session- 2022-23

Subject- Mathematics (Standard)

Sample Question Paper - 29

Time Allowed: 3 Hrs. **Maximum Marks : 80**

General Instructions:

1. This Question Paper has 5 Sections A-E.
2. Section **A** has 20 MCQs carrying 1 mark each
3. Section **B** has 5 questions carrying 02 marks each.
4. Section **C** has 6 questions carrying 03 marks each.
5. Section **D** has 4 questions carrying 05 marks each.
6. Section **E** has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

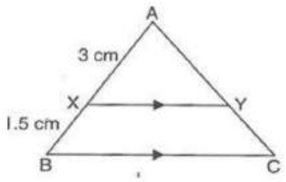
Section A

1. A polynomial of degree ________ is called a linear polynomial. **[1]**

a) 1 b) 3

c) 2 d) 0

2. In the given figure $XY||BC$. If AX = 3cm, XB = 1.5 cm and BC = 6cm, then XY is equal to **[1]**



a) 6 cm. b) 4.5 cm

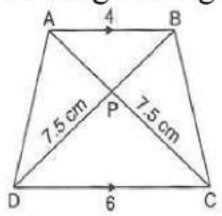
c) 3 cm. d) 4 cm.

3. The solution of 3x - 5y = -16 and 2x + 5y = 31 **[1]**

a) x = -3, y = 5 b) x = -3, y = -5

c) x = 3, y = -5 d) x = 3, y = 5

4. In the given figure, if AB||DC, then AP is equal to [1]



a) 5 cm. b) 7 cm.

c) 6 cm. d) 5.5 cm.

5. The difference between two numbers is 26 and one number is three times the other. The numbers are [1]

a) 39 and 13 b) 30 and 10

c) 36 and 12 d) 36 and 10

6. The probability of throwing a number greater than 2 with a fair dice is [1]

a) $\frac{1}{3}$ b) $\frac{2}{3}$

c) $\frac{3}{5}$ d) $\frac{2}{5}$

7. Consider the following frequency distribution: [1]

Class	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	3	9	15	30	18	5

The modal class is

a) 50-60 b) 20-30

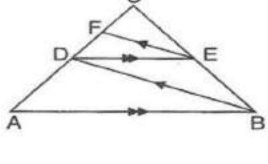
c) 10-20 d) 30-40

8. If angles A, B, C of a ΔABC form an increasing AP, then sin B = [1]

a) $\frac{1}{2}$ b) $\frac{1}{\sqrt{2}}$

c) 1 d) $\frac{\sqrt{3}}{2}$

9. In the given figure, $AB||DE$ and $BD||EF$. Then, [1]



a) $BC^2 = AB.CE$ b) $DC^2 = CF \times AC$

c) $AB^2 = AC.DE$ d) $AC^2 = BC.DC$

10. The sum of two numbers is 17 and the sum of their reciprocals is $\frac{17}{62}$. The quadratic representation of the above situation is **[1]**

a) $\frac{1}{x}+\frac{1}{x+17}=\frac{17}{62}$

b) $\frac{1}{x(17-x)}=\frac{17}{62}$

c) $\frac{1}{x}+\frac{1}{17-x}=\frac{17}{62}$

d) $\frac{1}{x}-\frac{1}{17-x}=\frac{17}{62}$

11. The number $(\sqrt{3}+\sqrt{5})^2$ is **[1]**

a) an irrational number

b) an integer

c) a rational number

d) not a real number

12. The mean of the data when $\sum f_i d_i = 435$, $\sum f_i = 30$ and a = 47.5 is **[1]**

a) 47.5

b) 62

c) 30

d) 63

13. The point which lies on the perpendicular bisector of the line segment joining the points A (-2, -5) and B (2, 5) is **[1]**

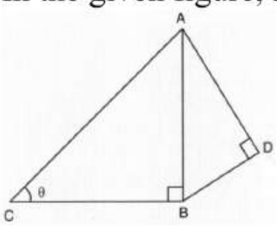
a) (2, 0)

b) (–2, 0)

c) (0, 2)

d) (0, 0)

14. In the given figure, if AD = 4 cm BD = 3 and CB = 12 cm, then cotθ is **[1]**



a) $\frac{13}{12}$

b) $\frac{12}{5}$

c) $\frac{12}{13}$

d) $\frac{5}{12}$

15. The angle of elevation of the top of a tower at a point on the ground 50 m away from the foot of the tower is 45°. Then the height of the tower (in metres) is **[1]**

a) $\frac{50}{\sqrt{2}}$

b) $\frac{50}{\sqrt{3}}$

c) $50\sqrt{3}$

d) 50

16. A circle is inscribed in a quadrilateral ABCD in which $\angle B = 90^o$, if AD = 23 cm, AB = 29 cm and DS = 5 cm, then radius of circle is : **[1]**

a) 12 cm

b) 13 cm

c) 14 cm

d) 11 cm

17. In $\triangle$DEF and $\triangle$PQR, it is given that $\angle$D = $\angle$Q and $\angle$R = $\angle$E, then which of the following is not true? **[1]**

a) $\frac{DE}{QR} = \frac{DF}{PQ}$

b) $\frac{EF}{RP} = \frac{DE}{QR}$

c) $\frac{EF}{PR} = \frac{DF}{PQ}$

d) $\frac{DE}{PQ} = \frac{EF}{RP}$

18. **Assertion (A):** A cubic polynomial may cut x-axis at one point and can touch at other points. **[1]**
Reason (R): Cubic polynomials have always two zeroes.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

19. The roots of a quadratic equation $x^2 - 4px + 4p^2 - q^2 = 0$ are **[1]**

a) 2p + q, 2p - q

b) p + 2q, p - 2q

c) 2p + q, 2p + q

d) 2p - q, 2p - q

20. **Assertion (A):** Two identical solid cubes of side 5 cm are joined end to end. The total surface area of the resulting cuboid is 350 cm^2. **[1]**
Reason (R): Total surface area of a cuboid is 2(lb + bh + hl)

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Find the roots of the quadratic equation $2x^2 - x + \frac{1}{8} = 0$ by factorization. **[2]**

22. Find the values of x for which the distance between the points P(x, 4) and Q(9,10) is 10 units. **[2]**

23. Show that $3\sqrt{2}$ is irrational. **[2]**

24. If the diagonal BD of a quadrilateral ABCD bisects both $\angle B$ and $\angle D$ prove that $\frac{AB}{BC} = \frac{AD}{CD}$. **[2]**

OR

Prove that the line joining the mid points of any two sides of a triangle is parallel to the third side.

25. Prove that: $\frac{1+\sec\theta-\tan\theta}{1+\sec\theta+\tan\theta} = \frac{1-\sin\theta}{\cos\theta}$ **[2]**

OR

Prove that: $\cot^4 A - 1 = \text{cosec}^4 A - 2\,\text{cosec}^2 A$

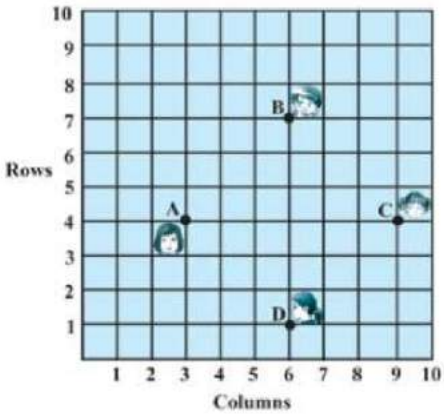
Section C

26. Solve: $\frac{x}{x-1} + \frac{x-1}{x} = 4\frac{1}{4}, x \neq 2, 0$ **[3]**

27. In a $\triangle ABC$, let D be a point on BC such that $\frac{BD}{DC} = \frac{AB}{AC}$ Prove that AD is the bisector o f $\angle A$. **[3]**

28. Show that one and only one out of n, (n + 2) or (n + 4) is divisible by 3, where n EN. **[3]**

29. Read the following passage and answer the question that follows: **[3]**
In a class room, four student Sita, Gita, Rita and Anita are sitting at A(3, 4), B(6, 7), C(9, 4), D(6, 1) respectively. Then a new student Anjali joins the class.



i. Teacher tells Anjali to sit in the middle of the four students. Find the coordinates of the position where she can sit.
ii. Calculate the distance between Sita and Anita.
iii. Which two students are equidistant from Gita.

OR

A point P divides the line segment joining the points A (3, - 5) and B (- 4, 8) such that $\frac{AP}{PB} = \frac{k}{1}$. If P lies on the line x + y = 0, then find the value of k.

30. Two ships are there in the sea on either side of a lighthouse in such a way that the ships and the lighthouse are in the same straight line. The angles of depression of two ships are observed from the top of the lighthouse are 60° and 45° respectively. If the height of the lighthouse is 200 m, find the distance between the two ships. (Use $\sqrt{3}$ = 1.73) **[3]**

OR

If the angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are complementary, find the height of the tower.

31. During a medical check-up, the number of heartbeats per minute of 30 patients were recorded and summarised as follows: **[3]**

Number of heartbeats per minute	65 - 68	68 - 71	71 - 74	74 - 77	77 - 80	80 - 83	83 - 86
Number of patients	2	4	3	8	7	4	2

Find the mean of heartbeats per minute for these patients, choosing a suitable method.

Section D

32. Form a pair of linear equations in two variables using the following information and solve it graphically. Five years ago, sagar was twice as old as Tiru. Sagar's age will be ten years more than Tiru's age. Find their present age. **[5]**

OR

Romila went to a stationary stall and purchased 2 pencils and 3 erasers for Rs.9. Her friend Sonali saw the new variety of pencils and erasers with Romila, and she also bought 4 pencils and 6 erasers of the same kind for Rs.18. Represent this situation algebraically and graphically.

33. A line drawn through the end point of a radius and perpendicular to it is a tangent to the circle. **[5]**

34. A bag contains 4 white balls, 6 red balls, 7 black balls and 3 blue balls. One ball is drawn at random from the bag. Find the probability that the ball drawn is **[5]**

i. white

ii. not black

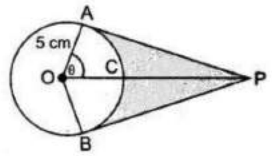
iii. neither white nor black

iv. red or white.

35. Four equal circles are described at the four corners of a square so that each touches two of the others. The shaded area enclosed between the circles is $\frac{24}{7}\text{cm}^2$. Find the radius of each circle. **[5]**

OR

An elastic belt is placed round the rim of a pulley of radius 5 cm. One point on the belt is pulled directly away from the centre O of the pulley until it is at P, 10 cm from O. Find the length of the belt that is in contact with the rim of the pulley. Also, find the shaded area.

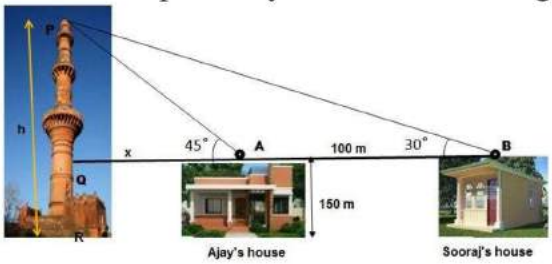


Section E

36. **Read the text carefully and answer the questions:** **[4]**

The houses of Ajay and Sooraj are at 100 m distance and the height of their houses is the same as approx 150 m. One big tower was situated near their house. Once both friends decided to measure the height of the tower. They measure the angle of elevation of the top of the tower from the roof of their houses. The angle of elevation of ajay's house to the tower and sooraj's house to the tower are 45^{o}

and 30^{o} respectively as shown in the figure.



(i) Find the height of the tower.

(ii) What is the distance between the tower and the house of Sooraj?

(iii) Find the distance between top of the tower and top of Sooraj's house?

OR

Find the distance between top of tower and top of Ajay's house?

37. **Read the text carefully and answer the questions:** **[4]**

The students of a school decided to beautify the school on an annual day by fixing colourful flags on the straight passage of the school. They have 27 flags to be fixed at intervals of every 2 metre. The flags are stored at the position of the middlemost flag. Ruchi was given the responsibility of placing the flags. Ruchi kept her books where the flags were stored. She could carry only one flag at a time.

(i) How much distance did she cover in pacing 6 flags on either side of center point?

(ii) Represent above information in Arithmetic progression

OR

What is the maximum distance she travelled carrying a flag?

(iii) How much distance did she cover in completing this job and returning to collect her books?

38. **Read the text carefully and answer the questions:** **[4]**

A juice seller is serving his customers using cylindrical container with radius 20cm and height 50cm. He serves juice into a glass as shown in Fig. The inner diameter of the cylindrical glass is 5 cm, but the bottom of the glass had a hemispherical raised portion which reduced the capacity of the glass.

(i) If the height of a glass was 10 cm, find the apparent capacity of the glass.

(ii) Also, find its actual capacity. (Use $\pi = 3.14$)

OR

How many glasses he serves if the container is full?

(iii) Find the capacity of the container in liter?

SOLUTION

Section A

1. **(a)** 1

   Explanation: A polynomial of degree 1 is called a linear polynomial. Example 4x + 3, 65y are linear polynomials.

2. **(d)** 4 cm.

   Explanation: Since $XY||BC$, then using Thales theorem,

   $\Rightarrow \frac{\mathrm{AX}}{\mathrm{AB}} = \frac{\mathrm{XY}}{\mathrm{BC}}$

   $\Rightarrow \frac{3}{4.5} = \frac{\mathrm{XY}}{6}$

   $\Rightarrow$XY = 4 cm

3. **(d)** x = 3, y = 5

   Explanation: Given: 3x - 5y = -16 ... (i)

   And 2x + 5y = 31 ... (ii)

   Adding eq. (i) and (ii), we get

   5x = 15

   $\Rightarrow$ x = 3

   Putting the value of x in eq. (ii), we get

   2(3) + 5y = 31

   $\Rightarrow$ 5y = 31 - 6

   $\Rightarrow$ y = 5

4. **(a)** 5 cm.

   Explanation: In triangles APB and CPD,

   $\angle$APB = $\angle$CPD [Vertically opposite angles] $\angle$BAP = $\angle$ACD [Alternate angles as $AB||CD$]

   $\therefore\Delta$APB $\sim$ ΔCPD [AA similarity]

   $\therefore \frac{\mathrm{AB}}{\mathrm{CD}} = \frac{\mathrm{CP}}{\mathrm{AP}}$

   $\Rightarrow \frac{4}{6} = \frac{\mathrm{AP}}{7.5}$

   $\Rightarrow$ AP = $\frac{7.5\times4}{6}$ = 5 cm

5. **(a)** 39 and 13

   Explanation: Let the two numbers be x and y

   According to question, x - y = 26 and x = 3y

   Putting the value of x in x - y = 26, we get,

   3y - y = 26

   $\Rightarrow$ y = 13 And x = 3 $\times$ 13 = 39

   Therefore, the two numbers are 13 and 39.

6. **(b)** $\frac{2}{3}$

   Explanation: $\because$ A dice has 6 numbers

   $\therefore$ n = 6

   Numbers greater than 2 are 3, 4, 5, 6

   $\therefore$ m = 4

   Probability = $\frac{m}{n} = \frac{4}{6} = \frac{2}{3}$

7. **(d)** 30-40

Explanation: Class having maximum frequency is the modal class.
Here, maximum frequency = 30
Hence, the modal class is 30 - 40.

8. **(d)** $\frac{\sqrt{3}}{2}$

Explanation: Given, ∠A, ∠B and $\angle C$ of a ΔABC are an increasing AP.
$\therefore \angle \mathrm{B}=\frac{\angle \mathrm{A}+\angle \mathrm{C}}{2}$
But $\angle \mathrm{A}+\angle \mathrm{B}+\angle \mathrm{C} = 180^{\mathrm{o}}$
$\therefore 2\angle B=\angle A+\angle C$
$\Rightarrow 2\angle \mathrm{B}+\angle \mathrm{B}=\angle \mathrm{A}+\angle \mathrm{B}+\angle \mathrm{C}=180^{\circ}$
$\Rightarrow 3\angle \mathrm{B}=180^{\circ} \Rightarrow \angle \mathrm{B}=\frac{180^{\circ}}{3}=60^{\circ}$
$\therefore$ sin B = sin 60^{o} = $\frac{\sqrt{3}}{2}$

9. **(b)** $DC^2=CF\times AC$

Explanation: In triangle ABC, using Thales theorem,
$\frac{\mathrm{DC}}{\mathrm{AC}}=\frac{\mathrm{CE}}{\mathrm{BC}}$ [Since AB||DE] ....(i)
In triangle BCD, using Thales theorem,
$\frac{\mathrm{CF}}{\mathrm{DC}}=\frac{\mathrm{CE}}{\mathrm{BC}}$ [Since BD||EF] ....(ii)
From eq.(i) and (ii),
$\frac{\mathrm{DC}}{\mathrm{AC}}=\frac{\mathrm{CF}}{\mathrm{DC}}$
$\Rightarrow \mathrm{DC}^2 = \mathrm{CF} \times \mathrm{AC}$

10. **(c)** $\frac{1}{x}+\frac{1}{17-x}=\frac{17}{62}$

Explanation: Let one number be x, As the sum of the numbers is 17, then the other number will be (17 - x). Their reciprocals will be $\frac{1}{x}$ and $\frac{1}{17-x}$.
$\therefore$ According to question, $\frac{1}{x}+\frac{1}{17-x}=\frac{17}{62}$

11. **(a)** an irrational number

Explanation: $\left(\sqrt{3}+\sqrt{5}\right)^2=\left(\sqrt{3}\right)^2+\left(\sqrt{5}\right)^2+2\times\sqrt{3}\times\sqrt{5}$
$=3+5+2\sqrt{15}$
$=8+2\sqrt{15}$
Here, $\sqrt{15}=\sqrt{3}\times\sqrt{5}$
Since $\sqrt{3}$ and $\sqrt{5}$ both are an irrational number. Therefore, $\left(\sqrt{3}+\sqrt{5}\right)^2$ is an irrational number.

12. **(b)** 62

Explanation: Mean = $(\overline{x})=a+\frac{\sum f_i d_i}{\sum f_i}$
$=47.5+\frac{435}{30}$
$= 47.5 + 14.5 = 62$

13. **(d)** (0, 0)

Explanation: As we know that, the perpendicular bisector of the any line segment divides the line segment into two equal parts i.e., the perpendicular bisector of the line segment always passes through the mid - point of the line segment.
As mid - point of any line segment which passes through the points
(x_1, y_1) and (x_2, y_2) is;

$= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

So mid - point of the line segment joining the points A (- 2, - 5) and B (2, 5) will be;

$= \left(\frac{-2+2}{2}, \frac{-5+5}{2}\right) = (0, 0)$

Hence, (0, 0) is the required point lies on the perpendicular bisector of the lines segment.

14. **(b)** $\frac{12}{5}$

Explanation: $BA = \sqrt{(AD)^2 + (BD)^2}$

$= \sqrt{(4)^2 + (3)^2}$

$= \sqrt{25} = 5$

$\therefore AC = \sqrt{(BC)^2 + (AB)^2}$

$= \sqrt{(12)^2 + (5)^2}$

$= \sqrt{144 + 25}$

$= \sqrt{169} = 13$

Hence,

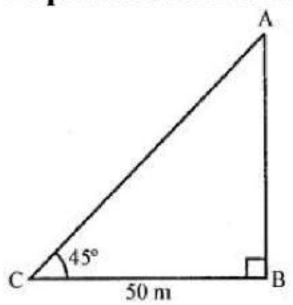
$\cos\theta = 12/13$

$and \sin\theta = 5/13$

$\Rightarrow \cot\theta = 12/5$

15. **(d)** 50

Explanation: Let AB be tower and C is as a point on the ground 52 m away



From foot of tower B

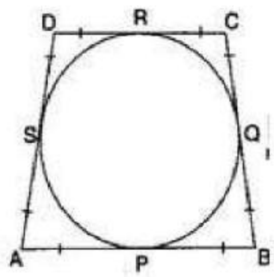
Angle of elevation is 45^{O}

let h be height of tower = x m

$\therefore \tan\theta = \frac{AB}{BC} \Rightarrow \tan 45^{O} = \frac{AB}{5}$

$\Rightarrow 1 = \frac{AB}{50} \Rightarrow = 50$ m

16. **(d)** 11 cm

Explanation: 

Here DS = DR = 5 cm

$\Rightarrow$ AS = 23 – 5 = 18 cm

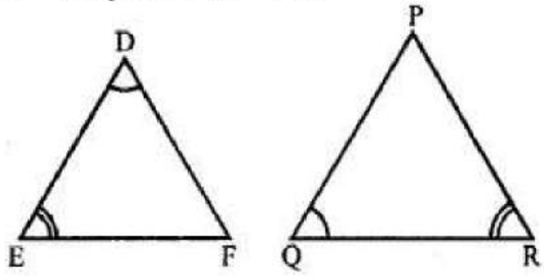
And AS = AP = 18 cm

$\Rightarrow$BP = 29 – 18 = 11 cm

$\because$ OP $\perp$ AB and OQ $\perp$ BC
$\therefore \angle$OQB = $\angle$OPB = 90° and $\angle$B = 90°
Also $\angle$POQ = 90°
Therefore, OPBQ is a square.
$\therefore$ BQ = OQ = 11 cm
Therefore Radius of circle = 11 cm

17. **(d)** $\frac{DE}{PQ} = \frac{EF}{RP}$

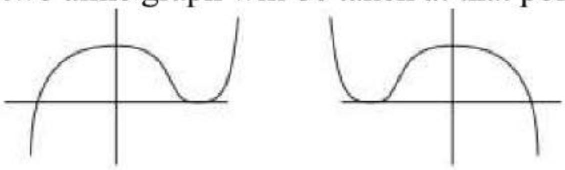
Explanation:
In $\triangle$DEF and $\triangle$PQR,
$\angle$D = $\angle$Q and $\angle$R = $\angle$E



Then $\frac{\text{DE}}{\text{PQ}} = \frac{\text{EF}}{\text{RP}}$ is not true.
($\because$ There are not corresponding sides)

18. **(c)** A is true but R is false.

Explanation:
Cubic polynomial can has zeroes like 2, 3 and 3, two alike and one distinct. Where two alike graph will be taken at that point and where distinct will cut like as under:



19. **(a)** 2p + q, 2p - q

Explanation: Given: $x^2 - 4px + 4p^2 - q^2 = 0$
$\Rightarrow (x - 2p)^2 - q^2 = 0$
Using $a^2 - b^2 = (a + b)(a - b)$,
$\Rightarrow (x - 2p + q)(x - 2p - q) = 0$
$\Rightarrow x - 2p + q = 0$ and $x - 2p - q = 0$
$\Rightarrow x = 2p - q$ and $x = 2p + q$

20. **(d)** A is false but R is true.

Explanation: A is false but R is true.

Section B

21. We have, $2x^2 - x + \frac{1}{8} = 0$
$\implies 2x^2 - \frac{1}{2}x - \frac{1}{2}x + \frac{1}{8} = 0$
$\implies x(2x - \frac{1}{2}) - \frac{1}{4}(2x - \frac{1}{2}) = 0$
$\implies (2x - \frac{1}{2})(x - \frac{1}{4}) = 0$
$Either\,(2x - \frac{1}{2}) = 0\,or\,(x - \frac{1}{4}) = 0$
$\implies x = \frac{1}{4}, \frac{1}{4}$
So, this root is repeated root.

$\therefore$ both the roots are $\frac{1}{4}$.

22. We have,

PQ = 10

$\Rightarrow PQ^2 = 100$

$\Rightarrow (9 - x)^2 + (10 - 4)^2 = 100$

$\Rightarrow (9 - x)^2 + 6^2 = 100$

$\Rightarrow (9 - x)^2 + 36 = 100$

$\Rightarrow (9 - x)^2 = 64$

$\Rightarrow 9 - x = \pm 8$

$\Rightarrow 9 - x = 8$ or $9 - x = -8$

$\Rightarrow x = 1$ or $x = 17$

Hence, the required values of x are 1 and 17.

23. Let us assume, to the contrary, that $3\sqrt{2}$ is rational.

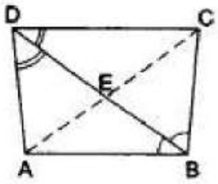
That is, we can find coprimes a and b ($b \neq 0$) such that $3\sqrt{2} = \frac{a}{b}$

Rearranging, we get $\sqrt{2} = \frac{a}{3b}$ ...(i)

Since 3, a and b are integers, $\frac{a}{3b}$ is rational, and so (i) shows that $\sqrt{2}$ is rational.

But this contradicts the fact that $\sqrt{2}$ is irrational.

So, we conclude that $3\sqrt{2}$ is irrational.

24. 

According to question it is given that ABCD is a quadrilateral in which diagonal BD bisects both $\angle B$ and $\angle D$.

To Prove $\frac{AB}{BC} = \frac{AD}{CD}$.

Construction Join AC, intersecting BD at E.

Proof: In $\triangle CBA$, BE is the bisector of $\angle ABC$

$\therefore \quad \frac{AE}{EC} = \frac{AB}{BC}$ ....(i) [using angle-bisector theorem]

In $\triangle ADC$, DE is the bisector of $\angle ADC$.

$\therefore \quad \frac{AE}{EC} = \frac{AD}{CD}$ ....(ii) [using angle-bisector theorem]

From equation (i) and equation (ii), we get

$\frac{AB}{BC} = \frac{AD}{CD}$.

OR

Given: A $\triangle ABC$ in which D and E are the mid-points of sides AB and AC respectively. DE is the line joining D and E.

To prove:DE $\parallel$ BC

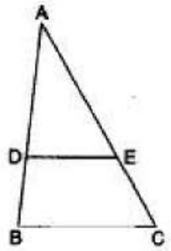
Proof:

$\because$ D is the mid-point of AB

$\therefore$ AD=DB

$\therefore \frac{AD}{DB} = 1$.....(1)

$\because$ E is the mid-point of AC



$\therefore$ AE=EC

$\therefore \frac{AE}{EC} = 1.....(2)$

From (1) and (2) $\frac{AD}{DB} = \frac{AE}{EC}$

$\therefore \frac{AE}{EC} = 1......(2)$....By converse of basic proportionality theorem

25. We have to prove that $\frac{1+\sec\theta-\tan\theta}{1+\sec\theta+\tan\theta} = \frac{1-\sin\theta}{\cos\theta}$

Recall identity $\sec^2\theta - \tan^2\theta = 1$

Here, LHS $= \frac{1+\sec\theta-\tan\theta}{1+\sec\theta+\tan\theta}$

$= \frac{\sec^2\theta-\tan^2\theta+\sec\theta-\tan\theta}{1+\sec\theta+\tan\theta}$

$= \frac{(\sec\theta-\tan\theta)(\sec\theta+\tan\theta)+(\sec\theta-\tan\theta)}{1+\sec\theta+\tan\theta}$ [because, $a^2 - b^2 = (a-b)(a+b)$]

$= \frac{(\sec\theta-\tan\theta)[\sec\theta+\tan\theta+1]}{(\sec\theta+\tan\theta+1)}$

$= \sec\theta - \tan\theta$

$= \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}$

$= \frac{1-\sin\theta}{\cos\theta}$ = RHS

Hence, proved.

OR

We have,

LHS = $\cot^4 A - 1$

$\Rightarrow$ LHS = $(\text{cosec}^2 A - 1)^2 - 1$ [$\because \cot^2 A = \text{cosec}^2 A - 1 \therefore \cot^4 A = (\text{cosec}^2 A - 1)^2$]

$\Rightarrow$ LHS = $\text{cosec}^4 A - 2\,\text{cosec}^2 A + 1 - 1$

$= \text{cosec}^4 A - 2\,\text{cosec}^2 A$ = RHS

Section C

26. The given equation is:

$\frac{x}{x-1} + \frac{x-1}{x} = 4\frac{1}{4}$

$\Rightarrow \frac{x}{x-1} + \frac{x-1}{x} = \frac{17}{4}$

put $\frac{x}{x-1}$ = y , we obtain

$y + \frac{1}{y} = \frac{17}{4}$

$\Rightarrow \frac{y^2+1}{y} = \frac{17}{4}$

$\Rightarrow$ 4y^2 + 4 = 17y

$\Rightarrow$ 4y^2 - 17y + 4 = 0

$\Rightarrow$ 4y^2 - 16y - y + 4 = 0

$\Rightarrow$ 4y(y - 4) -1(y - 4) = 0

$\Rightarrow$ (y - 4) (4y - 1) = 0

$\Rightarrow$ y - 4 = 0 or 4y - 1 = 0

Therefore, either y = 4 or y = $\frac{1}{4}$.

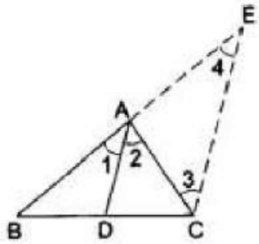
Now $\frac{x}{x-1}$ = y

$\Rightarrow \frac{x}{x-1} = 4$ or $\frac{x}{x-1} = \frac{1}{4}$

$\Rightarrow$ x = 4x - 4 or 4x = x - 1

$\Rightarrow$ 3x = 4 or 3x = -1

Hence the values of x are $x = \frac{4}{3}$ and $x = \frac{-1}{3}$.

27. 

It is given that in $\triangle ABC$, D is a point on BC such that $\frac{BD}{DC} = \frac{AB}{AC}$.

To Prove: AD is the bisector of $\angle A$.

Construction: Produce BA to E such that AE = AC and Join EC.

Proof: $\frac{BD}{DC} = \frac{AB}{AC}$ ....(given)

$\Rightarrow \quad \frac{BD}{DC} = \frac{AB}{AE}$.............................. [$\because$ AC = AE]

$\Rightarrow \quad DA \| CE$ [by the converse of Thales' theorem]

$\therefore \quad \angle 2 = \angle 3$ ....(i) [alternate interior angle]

and $\angle 1 = \angle 4$ ....(ii) [corresponding angle]

Also, AE = AC

$\Rightarrow \angle 3 = \angle 4$ ....(iii)

$\therefore \quad \angle 1 = \angle 2$ [from (i), (ii) and (iii)].

Hence, AD is the bisector of $\angle A$.

28. Let the number be (3q + r)

$n = 3q + r \quad 0 \leq r < 3$

or $3q, 3q + 1, 3q + 2$

If $n = 3q$ then, numbers are $3q$, $(3q + 1)$, $(3q + 2)$

$3q$ is divisible by 3.

If $n = 3q + 1$ then, numbers are $(3q + 1), (3q + 3), (3q + 4)$

$(3q + 3)$ is divisible by 3.

If $n = 3q + 2$ then, numbers are $(3q + 2), (3q + 4), (3q + 6)$

$(3q + 6)$ is divisible by 3.

$\therefore$ out of $n, (n + 2)$ and $(n + 4)$ only one is divisible by 3.

29. i. Given: A(3, 4), B(6, 7), C(9, 4), D(6, 1)

Using distance formula,

AB = $\sqrt{(6-3)^2 + (7-4)^2} = 3\sqrt{2}$ units

BC = $\sqrt{(9-6)^2 + (4-7)^2} = 3\sqrt{2}$ units

CD = $\sqrt{(6-9)^2 + (1-4)^2} = 3\sqrt{2}$ units

DA = $\sqrt{(3-6)^2 + (4-1)^2} = 3\sqrt{2}$ units

AC = $\sqrt{(9-3)^2 + (4-4)^2} = 6$ units

BD = $\sqrt{(6-6)^2 + (1-7)^2} = 6$ units

As sides AB = BC = CD = DA, and diagonals AC and BD are equal, so ABCD is a square.

Now as diagonals of a square bisect each other, so midpoint of the diagonal gives the position of Anjali to sit in the middle of the four students.

Here diagonal is AC or BD.

So, mid-point of AC = $\left(\frac{3+9}{2}, \frac{4+4}{2}\right)$ = (6, 4)

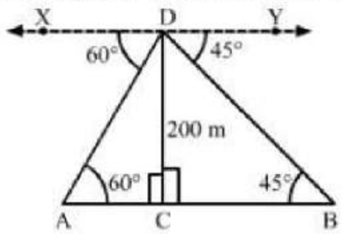
So, position of Anjali is (6, 4).

ii. Position of Sita is at point A i.e. (3, 4) and Position of Anita is at point D i.e. (6, 1).
So, distance between Sita and Anita, AD $=\sqrt{(6-3)^2+(1-4)^2}=3\sqrt{2}$ units

iii. Now, Gita is at position B and as BA and BC are equal and equidistant from point B.
So, we can say Sita and Rita are the two students who are equidistant from Gita.

OR

Given points are A(3, -5) and B(-4, 8).
P divides AB in the ratio k:1
Using the section formula, we have:
Coordinate of point P are $\left\{\left(\frac{-4k+3}{k+1}\right)\left(\frac{8k-5}{k+1}\right)\right\}$
Now it is given, that P lies on the line x + y =0
Therefore,
$\frac{-4k+3}{k+1}+\frac{8k-5}{k+1}=0$
$\Rightarrow$ -4k + 3 + 8k - 5 =0
$\Rightarrow$ -4k + 3 + 8k - 5 = 0
$\Rightarrow$ 4k - 2 = 0
$\Rightarrow k=\frac{2}{4}$
$\Rightarrow k=\frac{1}{2}$
Thus, the value of k is 1/2.

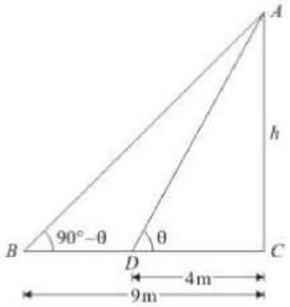
30. 

Let CD be the lighthouse and A and B be the positions of the two ships.
Height of the lighthouse, CD = 200 m
Now,
$\angle$CAD = $\angle$ADX = 60º (Alternate angles)
$\angle$CBD = $\angle$BDY = 45º (Alternate angles)
In right $\triangle$ACD,
$\tan 60^\circ=\frac{\text{CD}}{\text{AC}}$
$\Rightarrow \sqrt{3}=\frac{200}{\text{AC}}$
$\Rightarrow$ AC = $\frac{200}{\sqrt{3}}=\frac{200\sqrt{3}}{3}$ m
In right $\triangle$BCD,
$\tan 45^{\text{o}}=\frac{\text{CD}}{\text{BC}}$
$\Rightarrow 1=\frac{200}{BC}$
$\Rightarrow$ BC = 200 m
$\therefore$ Distance between the two ships, AB = BC + AC
= 200 + $\frac{200\sqrt{3}}{3}$
= 200 + $\frac{200\times1.73}{3}$
= 200 + 115.33

= 315.33 m (approx)
Hence, the distance between the two ships is approximately 315.33 m.

OR

Let AC be the height of tower is h meters.



Given that: angle of elevation are $\angle B = 90^{o} - \theta$ and $\angle D = \theta$ and also CD = 4 m and BC = 9 m.
Here we have to find height of tower.
So we use trigonometric ratios.
In a triangle ADC,
$\tan\theta = \frac{h}{4}$
Again in a triangle ABC,
$\Rightarrow \tan(90^{\circ} - \theta) = \frac{AC}{BC}$

$\Rightarrow \cot\theta = \frac{h}{9}$
$\Rightarrow \frac{1}{\tan\theta} = \frac{h}{9}$
Put $\tan\theta = \frac{h}{4}$
$\Rightarrow \frac{4}{h} = \frac{h}{9}$
$\Rightarrow h^2 = 36$
$\Rightarrow h = 6$
Hence height of tower is 6 meters.

31. Following table shows the given data & assumed mean deviation method to calculate the mean :-

Class Interval	**Frequency(f_i)**	**Mid value x_i**	Deviation $d_i = x_i - 75.5$	$(f_i \times d_i)$
65 - 68	2	66.5	-9	-18
68 - 71	4	69.5	-6	-24
71 - 74	3	72.5	-3	-9
74 - 77	8	75.5 = A	0	0
77 - 80	7	78.5	3	21
80 - 83	4	81.5	6	24
83 - 86	2	84.5	9	18
	$\Sigma f_i = 30$			$\Sigma(f_i d_i) = 12$

Let, assumed mean (A) = 75.5.....(1)

Now, from table :-
$\sum f_i = 30$ and $\sum f_i d_i = 12$......(2)

Now,
mean= $A + \frac{\sum f_i d_i}{\sum f_i}$
= 75.5 + $\frac{12}{30}$. [from (1) & (2)]
= 75.5 + 0.4
= 75.9
Thus, the mean of heartbeats per minute for these patients is 75.9

Section D

32. Let the present age of Sagar be x years and the age of Tiru be y year.
5 years ago, Sagar's age = $(x-5)$years and Tiru's age =$(y-5)$years
According to given condition,
$(x-5) = 2(y-5)$
$\Rightarrow x - 5 = 2y - 10 \Rightarrow x - 2y + 5 = 0$
After 10 yr, Sagar's age = $(x+10)$yrs and Tiru's age =$(y+10)$yrs
According to the given question,
$x + 10 = (y + 10) + 10$
$\Rightarrow x + 10 = y + 20 \Rightarrow x - y - 10 = 0$
Thus, we get following pair of linear equations
$\Rightarrow x - 2y + 5 = 0$ ....(i)
$\Rightarrow x - y - 10 = 0$ ...(ii)
Now, Let us draw the graphs of Eqs.(i) and (ii) , by finding atleast two solutions of each of the above equations. The solutions of equations are given in the following tables.

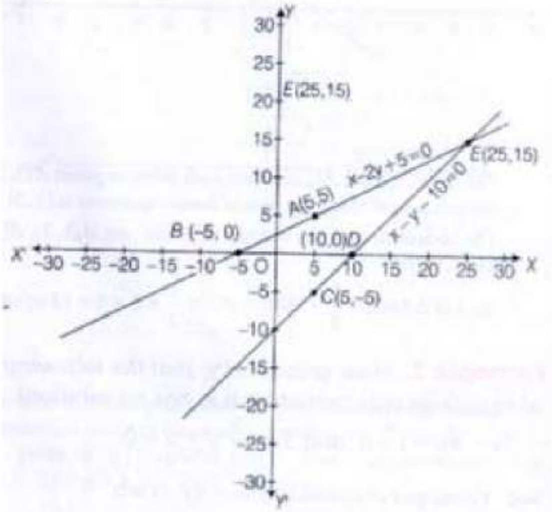
Table for $x - 2y + 5 = 0$

x	5	-5
$y = \frac{x+5}{2}$	5	0
Points	A(5,5)	B(-5,0)

Table for $x - y - 10 = 0$

x	5	10
y = x - 10	-5	0
Points	C(-5,5)	D(10,0)

Plot the points A(5,5) and B(-5,0) on a graph paper and join them to get the line AB. Similarly, plot the points C(-5,5) and D(10,0) on the same graph paper and join them to get line CD.



It is clear from the graph that, lines AB and CD intersect each other at point E(25,15).
So, $x = 25$ and $y = 15$ is the required solution.
Hence, Sagar's present age = 25 yr and Tiru's present age = 15 yr

OR

Formulation: Let the cost of 1 pencil be Rs. x and that of one eraser be Rs. y.
It is given that Romila purchased 2 pencils and 3 erasers for Rs.9.
2x + 3y = 9
It is also given that Sonali purchased 4 pencils and 6 erasers for Rs.18.
4x + 6y = 18
Algebraic Representation: The algebraic representation of the given situation is
2x + 3y = 9 ...(i)
4x + 6y = 18 ...(ii)

Graphical Representation: In order to obtain the graphical representation of the above pair of linear equations, we find two points on the line representing each equation. That is, we find two solutions of each equation. Let us find these solutions. We will try to find solutions having integral values. Therefore, We have,
x + 3y = 9
Putting x = - 3, we get
-6 + 3y = 9

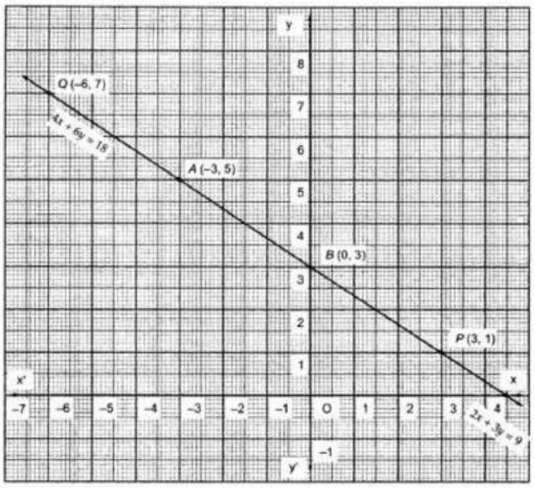
or, 3y = 15
or, y = 5
Putting x = 0, we get
0 + 3y = 9
or, y = 3
Thus, two solutions of 2x + 3y = 9 are:

x	-3	0
y	5	3

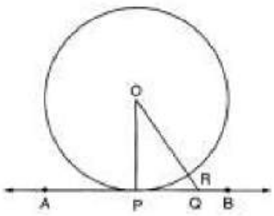
We have,
4x + 6y = 18
Putting x = 3, we get
4(3) + 6y = 18
12 + 6y = 18
⇒6y = 18 - 12
or, 6y = 6
or, y = 1
Putting x = - 6, we get
4(-6) + 6y = 18
-24 + 6y = 18
⇒ 6y = 18 + 24 = 42
or, y = 7
Thus, two solutions of 4x + 6y = 18 are

x	3	-6
y	1	7

Now, we plot the points A (-3, 5) and B (0, 3) and draw the line passing through these points to obtain the graph of the line 2x + 3y = 9. Points P (3,1) and Q (-6,7) are plotted on the graph paper and we join. Now, we plot the points A (-3, 5) and B (0, 3) and draw the line passing through these points to obtain the graph of the line 2x + 3y = 9. Points P (3,1) and Q (-6,7) are plotted on the graph paper and we join them to obtain the graph of the line 4x + 6y = 18. We find that both the lines AB and PQ coincide.



33. Given A radius OP of a circle C (O, r) and a line APB, perpendicular to OP.
To Prove AB is a tangent to the circle at the point P.



PROOF Take a point Q, different from P, on the line AB.since radius through the point of contact of tangent is perpendicular to it. Therefore,

$OP \perp AB.$

We know that among all the line segments joining O to a point on AB, OP is the shortest one. Therefore,

OP < OQ

$\Rightarrow$ OQ > OP

$\Rightarrow$Q lies outside the circle.

Therefore, every point on AB, other than P, lies outside the circle. This implies that AB meets the circle only at the point P.

Hence, AB is a tangent to the circle at P.

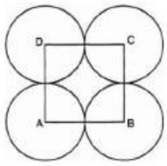
34. Number of white balls in the bag = 4
Number of red balls in the bag = 6
Number of black ball in the bag = 7
Number of blue balls in the bag = 3
∴ Total number of balls in the bag = 4 + 6 + 7 + 3 = 20
∴ Number of all possible outcomes = 20
$Proabibilty\ of\ the\ event = \frac{Number\ of\ favourble\ outcomes}{Total\ number\ of\ possible\ outcomes}$

i. Let E be the event that the ball drawn is white.
Then, the number of outcomes favourable to E is 4.
So, P(E) = P (white)= $\frac{4}{20} = \frac{1}{5}$

ii. Let E be the event that the ball drawn is no black.
Then, the number of outcomes favourable to E is 4 + 6 + 3 = 13.
So, P(E) = P(not black)=$\frac{13}{20}$

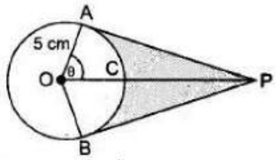
iii. Let E be the event that the ball drawn is neither white nor black.
Then, the number of outcomes favourable to E is 6 + 3 = 9.
So, P(E) = P (neither white nor black)= $\frac{9}{20}$

iv. Let E be the event that the ball drawn is red or white.
Then, the number of outcomes favourable to E is 6 + 4 = 10.
So, P(E) = P (red or white) = $\frac{10}{20} = \frac{1}{2}$

35. 

Let r cm be the radius of each circle.
Area of square - Area of 4 sectors = $\frac{24}{7}\text{cm}^2$
$(\text{side})^2$ - 4 $\left[\frac{\theta}{360}\pi \text{r}^2\right] = \frac{24}{7}\text{cm}^2$
or, $(2r)^2 - 4\left(\frac{90^\circ}{360^\circ} \times \pi r^2\right) = \frac{24}{7}$
or, $(2r)^2 - 4\left(\frac{1}{4^\circ} \times \pi r^2\right) = \frac{24}{7}$
or, $(2r)^2 - \left(\pi r^2\right) = \frac{24}{7}$
or, $4r^2 - \frac{22}{7}r^2 = \frac{24}{7}$
or, $\frac{28r^2 - 22r^2}{7} = \frac{24}{7}$
or, $6r^2 = 24$
or, $r^2 = 4$
or, $r = \pm 2$
or, Radius of each circle is 2 cm (r cannot be negative)

OR



$\cos\theta = \frac{1}{2}$ or, $\theta = 60^\circ$

Reflex $\angle AOB = 120^\circ$

$\therefore$ ADB = $\frac{2\times3.14\times5\times240}{360}$ = 20.93 cm

Hence length of elastic in contact = 20.93 cm

Now, AP = $5\sqrt{3}$cm

a ($\triangle$OAP) = $\frac{1}{2}\times$ base $\times$ height = $\frac{1}{2}\times 5\times 5\sqrt{3} = \frac{25\sqrt{3}}{2}$

Area ($\triangle$OAP + $\triangle$OBP) = 2$\times$ $\frac{25\sqrt{3}}{2} = 25\sqrt{3}$ = 43.25 cm^2

Area of sector OACB = $\frac{\theta}{360}\times \pi r^2$

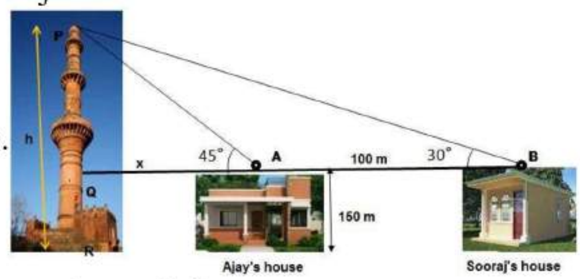
$= \frac{25\times3.14\times120}{360}$ = 26.16 cm^2

Shaded Area = 43.25 - 26.16 = 17.09 cm^2

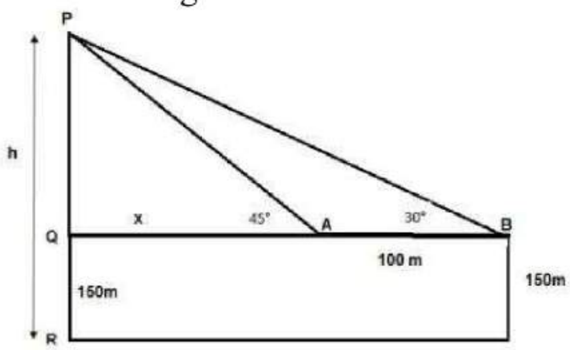
Section E

36. **Read the text carefully and answer the questions:**

The houses of Ajay and Sooraj are at 100 m distance and the height of their houses is the same as approx 150 m. One big tower was situated near their house. Once both friends decided to measure the height of the tower. They measure the angle of elevation of the top of the tower from the roof of their houses. The angle of elevation of ajay's house to the tower and sooraj's house to the tower are 45^o and 30^o respectively as shown in the figure.



(i) The above figure can be redrawn as shown below:



Let PQ = y

In $\triangle$PQA,

tan 45 = $\frac{PQ}{QA} = \frac{y}{x}$

$1 = \frac{y}{x}$

x = y ...(i)

In $\triangle$PQB,

$\tan 30 = \frac{PQ}{QB} = \frac{PQ}{x+100} = \frac{y}{x+100} = \frac{x}{x+100}$

$\frac{1}{\sqrt{3}} = \frac{x}{x+100}$

$x\sqrt{3} = x + 100$

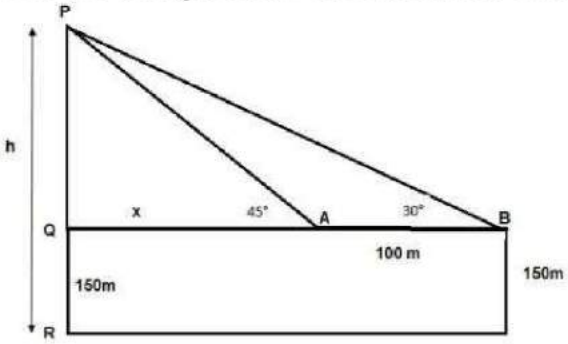
$x = \frac{100}{\sqrt{3}-1} = 136.61$ m

From the figure, height of tower h = PQ + QR

= x + 150 = 136.61 + 150

h = 286.61 m

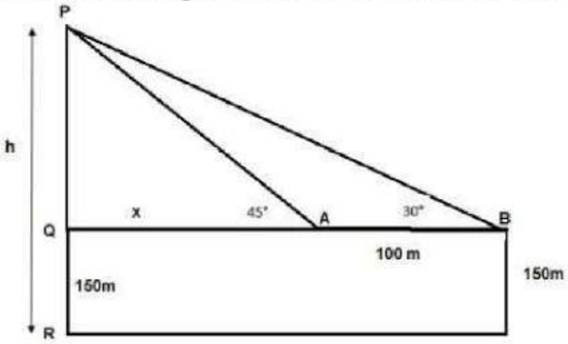
(ii) The above figure can be redrawn as shown below:



Distance of Sooraj's house from tower = QA + AB

= x + 100 = 136.61 + 100 = 236.61 m

(iii)The above figure can be redrawn as shown below:



Distance between top of tower and Top of Sooraj's house is PB

In $\triangle$PQB

$\sin 30^{O} = \frac{PQ}{PB}$

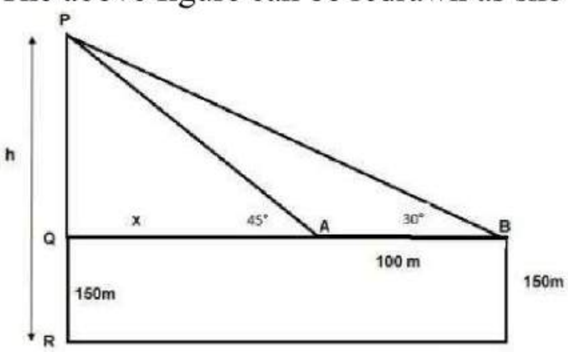
$\Rightarrow PB = \frac{PQ}{\sin 30^{o}}$

$\Rightarrow$ PB $= \frac{y}{\frac{1}{2}} = 2 \times 136.61$

$\Rightarrow$ PB = 273.20 m

OR

The above figure can be redrawn as shown below:



Distance between top of the tower and top of Ajay's house is PA

In $\triangle$PQA

$\sin 45^o = \frac{PQ}{PA}$

$\Rightarrow PA = \frac{PQ}{\sin 45^o}$

$\Rightarrow PA = \frac{y}{\frac{1}{\sqrt{2}}} = \sqrt{2} \times 136.61$

$\Rightarrow PA = 193.20$ m

37. **Read the text carefully and answer the questions:**

The students of a school decided to beautify the school on an annual day by fixing colourful flags on the straight passage of the school. They have 27 flags to be fixed at intervals of every 2 metre. The flags are stored at the position of the middlemost flag. Ruchi was given the responsibility of placing the flags. Ruchi kept her books where the flags were stored. She could carry only one flag at a time.

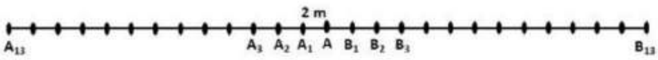
(i) Distance covered in placing 6 flags on either side of center point is 84 + 84 = 168 m

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$\Rightarrow S_6 = \frac{6}{2}[2 \times 4 + (6 - 1) \times 4]$

$\Rightarrow S_6 = 3[8 + 20]$

$\Rightarrow S_6 = 84$

(ii) 

Let A be the position of the middle-most flag.

Now, there are 13 flags ($A_1, A_2 ... A_{12}$) to the left of A and 13 flags ($B_1, B_2, B_3 ... B_{13}$) to the right of A.

Distance covered in fixing flag to $A_1 = 2 + 2 = 4$ m

Distance covered in fixing flag to $A_2 = 4 + 4 = 8$ m

Distance covered in fixing flag to $A_3 = 6 + 6 = 12$ m

...

Distance covered in fixing flag to $A_{13} = 26 + 26 = 52$ m

This forms an A.P. with,

First term, a = 4

Common difference, d = 4

and n = 13

OR

Maximum distance travelled by Ruchi in carrying a flag

= Distance from A_{13} to A or B_{13} to A = 26 m

(iii) $\because$ Distance covered in fixing 13 flags to the left of A = S_{13}

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$\Rightarrow S_{13} = \frac{13}{2}[2 \times 4 + 12 \times 4]$

$= \frac{13}{2} \times [8 + 48]$

$= \frac{13}{2} \times 56$

$= 364$

Similarly, distance covered in fixing 13 flags to the right of A = 364

Total distance covered by Ruchi in completing the task

$= 364 + 364 = 728$ m

38. **Read the text carefully and answer the questions:**

A juice seller is serving his customers using cylindrical container with radius 20cm and height 50cm. He serves juice into a glass as shown in Fig. The inner diameter of the cylindrical glass is 5 cm, but the bottom of the glass had a hemispherical raised portion which reduced the capacity of the glass.

(i) We have, Inner diameter of the glass, d = 5 cm, Height of the glass = 10 cm

The apparent capacity of the glass = Volume of cylinder

$= \pi r^2 h$

$= 3.14 \times \left(\frac{5}{2}\right)^2 \times 10$

$= 3.14 \times \frac{25}{4} \times 10 = 196.25 \text{ cm}^3$

(ii) We have, Inner diameter of the glass, d = 5 cm, Height of the glass = 10 cm

The actual capacity of glass = Apparent capacity of glass - Volume of hemispherical part of the glass

The volume of hemispherical part $= \frac{2}{3}\pi r^2 h = \frac{2}{3} \times 3.14 \times \left(\frac{5}{2}\right)^3 = 32.71 \text{ cm}^3$

Actual capacity of glass = 196.25 - 32.71 = 163.54 cm^3

OR

We have, Inner diameter of the glass, d = 5 cm, Height of the glass = 10 cm

Number of glasses $= \frac{\text{Volume of container}}{\text{Actual volume of one glass}}$

$\Rightarrow$ Number of glasses $= \frac{20 \times 3.14 \times 20 \times 50}{3.14 \times \frac{2\pi}{4} \times 10 - \frac{2}{3} \times 3.14 \times \frac{125}{8}}$

$\Rightarrow$ Number of glasses $= \frac{20000}{\frac{250}{4} - \frac{125}{12}} = \frac{20000 \times 12}{750 - 125} = \frac{240000}{625} = 384$

$\Rightarrow$ Number of glasses = 384

(iii) We have, inner diameter of the glass, d = 5 cm, height of the glass = 10 cm

Volume of container = V = $\pi r^2 h$

$\Rightarrow V = 3.14 \times 20 \times 20 \times 50 = 62800 \text{ cm}^3$

$\Rightarrow$ V = 62.8 litre