Amrita VISHWA VIDYAPEETHAM

(University established u/s 3 of UGC Act 1956)

Amrita Entrance Examination – Engineering

PHYSICS, CHEMISTRY & MATHEMATICS

Question booklet **Version Code**		Question booklet no.		Time : 3 hrs
Number of pages		Number of questions	**120**	Max. Marks : 360
Registration number				
Name of the candidate				
Signature of the candidate				

INSTRUCTIONS TO THE CANDIDATES

GENERAL

1. Any malpractice or attempt to commit malpractice in the examination hall will lead to disqualification of the candidate.
2. Candidates are not allowed to carry any textual material, printed or written bits of papers, Mathematical and Physical Tables, electronic gadgets like calculator, cell phone, etc. into the examination hall.
3. Candidates shall possess the University Hall Ticket which should be produced on demand.
4. Candidates shall occupy the respective seats bearing their registration numbers on time.
5. Candidates shall sign the attendance sheet available with the invigilator.
6. **Candidates are not permitted to leave the hall before the end of the examination.**
7. **Candidates are required to handover the ANSWER SHEET and the QUESTION BOOKLET to the invigilator before leaving the hall.**
8. *After submitting the answer sheet, candidates shall affix their left thumb impression on the attendance sheet.*

QUESTION BOOKLET

9. **DO NOT OPEN THE SEALED QUESTION BOOKLET UNTIL THE INVIGILATOR ANNOUNCES TO DO SO.**
10. **Before opening the Question Booklet,** write the Registration Number, Name and Signature using ball pen in the space provided at the top of this page.
11. **Immediately after opening the booklet,** the candidate should examine whether it contains all the 120 questions in serial order and ---- pages as mentioned at the top of this page. In case of unprinted, torn or missing pages in the booklet, the matter should be reported to the invigilator immediately.
12. Rough work may be done on the space provided in this booklet.

(Continued on the last page of this question booklet)

MODEL QUESTIONS

PHYSICS (S.No.1 to 35) 35 Questions

Data:
Acceleration due to gravity = 10m/s^2, Mass of electron = 9.1 x 10^{-31}kg
Charge of electron = 1.6 x 10^{-19} C, Velocity of light, c = 3 x 10^{8}m/s
1 eV = 1.6 x 10^{-19} J

1. Which of the following has the dimensionality of farad?

   a) $A^2s^4kg^{-1}m^{-2}$ b) $A^{-2}kg\ m^2s^3$ c) $kg\ m^2\ A^{-1}s^2$ d) $kg\ m^3\ A^{-2}s^2$

2. Choose the correct combination of the planet and its average orbital speed(in km s^{-1})

   a) Earth (29.8); Saturn(9.65); Venus(35.0); Mars(24.2)
   b) Earth (9.65); Saturn(29.8); Venus(35.0); Mars(24.2)
   c) Earth (24.2); Saturn(9.65); Venus(35.0); Mars(29.8)
   d) Earth (29.8); Saturn(9.65); Venus(24.2); Mars(35.0)

3. At a point 3200 km vertically above the surface of the earth, acceleration due to gravity of earth in SI units is

   a) 6.66   b) 3.33   c) 5.55   d) 4.44

4. Two laser beams one of wave length 640 nm and the other 400 nm have same unit flux of photons. Their powers are in the ratio

   a) 64:40   b) 1:1   c) 5:8   d) 25:64

5. The relation, Work Done = Change in internal energy holds for

   a) isothermal process   b) adiabatic process
   c) isobaric process   d) isochoric process

6. The rate of flow of volume of a fluid of viscosity η along a horizontal pipe of radius r and length L due to pressure difference ΔP is ($\Delta V/\Delta t$). If a pipe of radius 2r and length 2L is used and ΔP is doubled the rate of flow will increase by a factor

   a) 2   b) 4   c) 8   d) 16

7. If the charge Q in a capacitor is doubled, electric field energy stored inside

   a) doubles   b) increases by factor 4
   c) remains unchanged   d) increases by factor 8

8. A capacitor with C =0.144 μF having charge Q is made to discharge through a resistance of 1.0 Ω. What is the time taken for the discharge of 50% of the initial charge?

   a) 10^{-7} s   b) 0.144 x10^{-6} s   c) 2.1 x 10^{-7}s   d) 0.144 x 10^{-7} s

Rough Work

9. A slab having dielectric constant $\kappa = 3$ is placed in a region having constant electric field $E = 10$ V m^{-1}. The electric field inside the slab volume is

   a) 1.1 V m^{-1} b) 30 V m^{-1} c) zero d) 3.33V m^{-1}

10. A parallel plate capacitor is connected to a battery supplying constant voltage difference such that it accumulates charge Q. While being connected, if the separation d between the plates is increased

    a) both electric field inside the capacitor and Q decrease
    b) electric field inside the capacitor decreases and Q increases
    c) electric field inside the capacitor increases and Q decreases
    d) both electric field inside the capacitor and Q increase

11. The sides (in meters) of a box joining at origin are represented by vectors **a**= 4**i**, **b**= 2**i** + 3**j** and **c**= **i**+**k**. The surface area of the box is

    a) 20 m^2 b) 26 m^2 c) 36 m^2 d) 40 m^2

12. The slant side of a frictionless incline making an angle $60°$ with the vertical is 1 m. Starting from rest the time taken by a mass to slide down the incline from top to the base is

    a) 0.63 s b) 0.23 s c) 0.2 s d) 0.4 s

13. A mass of 0.01 kg is hung from a series combination of two ideal light springs having spring constants k_1= 10 Nm^{-1} and k_2 =20 Nm^{-1}. The net stretching of this spring-mass system is

    a) 3 cm b) 1.5 cm c) 6 cm d) 2.5 cm

14. A mass m =1 kg located at point (3,4) in x-y plane at time t is subjected to a force of 2 N in the y direction. All numbers are in SI units. The angular acceleration is

    a) 0.24 radians s^{-2} along z direction   b) 0.18 radians s^{-2} along z direction
    c) 0.12 radians s^{-2} along x direction   d) 0.32 radians s^{-2} along z direction

15. A circuit is operated by a battery of internal resistance 0.2 Ω and emf 6 V. The current flowing in the circuit is 0.3 A. The power supplied to the rest of the circuit other than the internal resistance is

    a) 1.8 W b) 1.74 W c) 1.42 W d) 1.62 W

16. A small magnet of magnetic moment **m** is placed inside a hollow sphere of radius R; the net magnetic flux emerging out of the sphere is

    a) proportional to **m**
    b) proportional to the product R^2 and magnitude of **m**
    c) zero
    d) a function of location and orientation of the magnet

Rough Work

17. What is the magnetic induction flux crossing unit area in xy plane if magnetic induction vector is $B = 2\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}$? All numbers are in SI units.

a) 2   b) 4   c) 6   d) $\sqrt{56}$

18. The direction of a ray of light from a plane wave is along unit vector $\mathbf{n} = \mathbf{i} + \mathbf{j}$. The corresponding wave front is

a) parallel to z axis   b) parallel to $\mathbf{n}$
c) perpendicular to z axis   d) parallel to y-z plane

19. Electric potential in a region is given by $4x^2+3$. All numbers are in SI units. The Electric field magnitude at a point (-5,1,2) is

a) 40   b) 20   c) 80   d) 10

20. When a glass prism of refracting angle 60° is immersed in a liquid, its angle of minimum deviation is 30°. The critical angle of glass with respect to the liquid medium is

a) 45° b) 30° c) 60° d) 55°

21. Choose the group of incorrect statement formed from the following
(i) The ammeter used to measure current in a circuit is to be connected in series.
(ii) An ammeter should have very low resistance.
(iii) An ammeter should have very high resistance.
(iv) Connecting ammeter in series will not lead to any change in the current present before.

a) (i) and (ii)   b) (ii) and (iii)   c) (iii) and (iv)   d) (iv) and (i)

22. Let E_i, N_i, I_i with $i=1,2$ denote respectively the emf, number of turns , and the current in the primary and secondary coils of an ideal transformer. Then

a) $E_1/E_2 = N_1/N_2 = I_1/I_2$ b) $E_1/E_2 = N_2/N_1 = I_1/I_2$
c) $E_2/E_1 = N_1/N_2 = I_1/I_2$ d) $E_1/E_2 = N_1/N_2 = I_2/I_1$

23. Which of the following are unrelated?
a) Fermat's principle and propagation of light
b) Huygen's principle and speed of light
c) Law of gravitation and Kepler's laws
d) Alpha decay and Coulomb force

24. A tiny electric dipole of dipole moment p $\mathbf{k}$ is placed at the origin. The electric fields at two far away point (b,0,0) and (0,0,b) are

a) equal in magnitude
b) equal
c) equal in direction only
d) unequal in magnitude and opposite in direction

Rough Work

25. A compound telescope have two lenses A and B. Lens A is closer to object than lens B. Which statement is correct?
    a) Both A and B form real images.
    b) Both A and B form virtual images.
    c) A forms real image and B forms virtual image.
    d) A forms virtual image and B forms real image.

26. Assume that the wave length of yellow light in crown glass of refractive index 1.5 is 600 nm. Its frequency is
    a) 0.5×10^{15} Hz
    b) 0.33×10^{15} Hz
    c) 1.5×10^{15} Hz
    d) 0.5×10^{15} Hz

27. The energies of two photons are in the ratio 1:4. The corresponding ratio of their momenta is
    a) 1:2
    b) 1:4
    c) 2:1
    d) 4:1

28. At a given kinetic energy which of the following has the highest speed?
    a) neutrino
    b) electron
    c) muon
    d) photon

29. The time taken by light to travel over a length equal to the radius of nucleus ^{64}Ni is of the order of
    a) 10^{-21} s
    b) 10^{-23} s
    c) 10^{-25} s
    d) 10^{-19} s

30. Water in a porcelain container is placed in a microwave oven to heat it. The temperature of the water rises, but the container temperature does not rise much. This is because
    a) porcelain is a bad conductor of heat.
    b) water is a liquid and can set up convection currents but the container is solid non conductor.
    c) preferential absorption of microwaves of certain frequencies by water.
    d) microwaves are more energetic than infrared waves.

IN COMPLETE

Rough Work

CHEMISTRY (S.No. 36 to 70) 35 Questions

36. 20 g of a solute whose density is 2.0 g/cc is dissolved in water and the solution is made upto one litre. If the molecular weight of the solute is 100, what is the molality of the solution?

    a) 0.2020　　　　　b) 0.4040　　　　　c) 0.2000　　　　　d) 0.0200

37. The velocity of infra red radiation in vacuum compared to ultra violet is

    a) twice　　　　　b) half　　　　　c) equal　　　　　d) four times

38. Which one of the following statements is true?

    a) An orbit and orbital mean the same thing.
    b) An orbit and orbital contain the same number of electrons always.
    c) The energies of the orbit and the orbital are the same.
    d) The maximum number of electrons present in an orbit and an orbital will be different.

39. Which one of the following has electronic configuration in violation of Aufbau principle?

    a) calcium　　　　　b) titanium　　　　　c) chromium　　　　　d) manganese

40. Which one of the following changes is spontaneous?

    a) A matchstick on strike burns.
    b) Camphor packed in a container without over space catches fire on its own.
    c) Petrol kept in an open beaker reduces in quantity slowly.
    d) Water in a beaker surrounded by ice and salt freezes.

41. For a substance A_2B the first dissociation constant is 5×10^{-5} and the second dissociation constant is 1×10^{-9} at $25^{\circ}C$. The value of the equilibrium constant for the following reaction

    $$A_2B \leftrightarrow 2A^+ + B^{2-}$$

    at the same temperature is

    a) 5×10^4　　　　　b) 2×10^{-5}　　　　　c) 4×10^{-4}　　　　　d) 5×10^{-14}

42. In ice-liquid water equilibrium, increase of pressure leads to

    a) increase in melting point of ice　　　　　b) decrease in melting point of ice
    c) no change in melting point of ice　　　　　d) disappearance of one phase

43. A silver rod dipped in a solution of silver nitrate of a particular concentration shows a potential of 0.75 V vs standard hydrogen electrode. If the standard potential for silver is 0.8V, at what molar concentration of the solution the potential will become zero?

    a) 2.76×10^{-14}　　　　　b) 2.76×10^{14}　　　　　c) 7.6×10^{-28}　　　　　d) 7.6×10^{28}

Rough Work

44. What is the theoretical quantity of hydrogen required to generate 53.6Ah in a Proton Exchange Membrane Fuel Cell?

a) 1.0 g   b) 1.0 kg   c) 2.0 g   d) 2.0 litre

45. For a reaction, $X + Y + Z \rightarrow$ Products
the concentration of X is doubled keeping that of Y and Z constant. The rate of the reaction increases by four times. What is the order of the reaction?

a) 2   b) 4   c) 1   d) 0

46. Which one of the following exhibits Schottky defect?

a) nickel oxide   b) potassium bromide
c) ferrous sulphide   d) silver chloride

47. Which one of the following is anti ferromagnetic?

a) titanium dioxide   b) nickel
c) oxygen   d) ferrous oxide

48. The gas that is produced through catalytic reforming of sewage is

a) producer gas   b) syngas
c) natural gas   d) carbon monoxide

49. Which one of the following hydrides is non-stoichiometric?

a) ammonia   b) nickel hydride   c) sodium hydride   d) diborane

50. The order of energy released on combustion of the following fuels per litre is

a) LPG > octane > liquid hydrogen > gaseous hydrogen
b) liquid hydrogen > gaseous hydrogen > LPG > octane
c) octane > LPG > liquid hydrogen > gaseous hydrogen
d) gaseous hydrogen > liquid hydrogen > octane > LPG

51. Density of the following alkali metals is in the order of

a) lithium < sodium < potassium < rubidium
b) rubidium < potassium < sodium < lithium
c) sodium < potassium < lithium < rubidium
d) lithium < potassium < sodium < rubidium

52. The discontinuity in ionization enthalpy values of group 13 elements in the periodic table is due to

a) irregular variation in ionic radii
b) irregular variation in electronegativity
c) poor shielding effect of 'p' and 'd' electrons
d) poor shielding effect of 'd' and 'f' electrons

Rough Work

53. The reduction of germanium tetrachloride with lithium aluminium hydride gives
    a) digermane
    b) di and tri germanes
    c) monogermane
    d) mixture of all germanes

54. Which one of the following is used as cathode in lithium primary battery?
    a) liquid sulphur dioxide
    b) thionyl chloride
    c) poly ethylene oxide
    d) methyl cyanide

55. What type of isomerism is possible in pentaamminenitrocobalt(II)chloride?
    a) linkage   b) optical   c) position   d) ionisation

56. A coordination compound has trigonal bipyramidal distribution of hybrid orbitals. What is the type of hybridisation present?
    a) dsp^2 b) sp^3 c) sp^3d d) d^2sp^3

57. 0.3 g of an organic compound gave 60 mL of nitrogen collected over water at 730 mm pressure and 27^oC. Aqueous tension at 27^oC is 20 mm. What is the percentage composition of nitrogen in the compound?
    a) 21.25   b) 2.125   c) 212.5   d) 42.5

58. Predict the products formed on passing acetylene through acetic acid followed by distillation in presence of mercuric sulphate.
    a) acetic anhydride and acetone
    b) acetic anhydride and ethanol
    c) propionic anhydride and methanol
    d) acetic anhydride and ethanal

59. The order of reactivity of the following for an S_N^2 reaction is
    a) alkyl fluoride > alkyl chloride > alkyl bromide > alkyl iodide
    b) alkyl fluoride > alkyl bromide > alkyl chloride > alkyl iodide
    c) alkyl iodide   > alkyl bromide > alkyl chloride > alkyl fluoride
    d) alkyl bromide > alkyl fluoride > alkyl iodide   > alkyl chloride

60. An organic compound A of molecular formula C_3H_8O is treated with 85% phosphoric acid at 170^oC to give B which on ozonolysis, followed by hydrolysis with lithium aluminium hydride gave rise to a set of products. Predict the correct set of products from the following
    a) acetaldehyde and formaldehyde
    b) ethanol and methanol
    c) acetic acid and formic acid
    d) ethanol and formaldehyde

IN COMPLETE

Rough Work

MATHEMATICS(S.No. 71 to 120) 50 Questions

71. Let $z_1 = 10 + 6i$ and $z_2 = 4 + 6i$. If z is any complex number such that the argument of $\dfrac{z - z_1}{z - z_2}$ is $\dfrac{\pi}{4}$, then $|z - 7 - 9i|$ is equal to

    (a) 6   (b) $3\sqrt{2}$ (c) $2\sqrt{3}$ (d) $\sqrt{6}$

72. The complex numbers z_1 and z_2 are such that $z_1 \neq z_2$ and $|z_1| = |z_2|$. If z_1 has positive real part and z_2 has negative imaginary part, then $\dfrac{z_1 + z_2}{z_1 - z_2}$ may be

    (a) zero   (b) real and negative

    (c) purely imaginary   (d) real and positive

73. The maximum value of $|z|$ where 'z' satisfies the condition $\left| z + \dfrac{2}{z} \right| = 2$ is

    (a) $\sqrt{3} - 1$ (b) $\sqrt{3}$ (c) $\sqrt{3} + 1$ (d) $\sqrt{3} + \sqrt{2}$

74. If 'ω' is a non real cube root of unity, then $(a + b)(a + b\omega)(a + b\omega^2)$ is

    (a) $a^2 - b^2$ (b) $a^3 + b^3$ (c) $a^3 - b^3$ (d) $a^2 + b^2$

75. If $a^2 + b^2 + c^2 = 1$, then, $bc + ca + ab$ lies in the interval

    (a) $\left[-\dfrac{1}{2}, \ 1 \right]$ (b) $\left[-\dfrac{1}{2}, \ 3 \right]$ (c) $[-1, \ 2]$ (d) $\left[-1, \ \dfrac{1}{2} \right]$

76. Let T_n denote the number of triangles which can be formed by using the vertices of a regular polygon of n sides. If $T_{n+1} - T_n = 10$, then the value of n is

    (a) 5   (b) 4   (c) 6   (d) 7

77. If $(2n + 1)P_{n-1} : (2n - 1)P_n = 3 : 5$, then the value of n is

    (a) 3   (b) 6   (c) 4   (d) 8

78. The inverse of the function $y = \dfrac{10^x - 10^{-x}}{10^x + 10^{-x}}$ is

    (a) $\log_{10}(2 - x)$ (b) $\dfrac{1}{2}\log_{10}\left(\dfrac{1 + x}{1 - x} \right)$

    (c) $\dfrac{1}{2}\log_{10}(2x - 1)$ (d) $\dfrac{1}{2}\log_{10}\left(\dfrac{2x}{2 - x} \right)$

Rough Work

79. The sum of the first n terms of the series $\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+\cdots\cdots\cdots$ is

(a) 2^n-1 　　　　(b) $1-2^{-n}$ 　　　　(c) $2^{-n}-n+1$ 　　　(d) $2^{-n}+n-1$

80. If $5^{1+x}+5^{1-x}$, $\frac{a}{2}$ and 25^x+25^{-x} are three consecutive terms of an A.P., then the values of 'a' are given by

(a) $a\geq 12$ 　　　　(b) $a>12$ 　　　　(c) $a<12$ 　　　　(d) $a\leq 12$

81. If $a,$ $b,$ c are in H.P., then the value of $\frac{b+a}{b-a}+\frac{b+c}{b-c}$ is

(a) 0 　　　　(b) 1 　　　　(c) 2 　　　　(d) 3

82. Let $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}=\Delta$, where $a,$ $b,$ c are positive. Then

(a) $\Delta>0$ 　　　　(b) $\Delta\geq 0$ 　　　　(c) $\Delta\leq 0$ 　　　　(d) $\Delta<0$

83. If $\begin{bmatrix} 1 & x & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 6 & 7 \end{bmatrix}\begin{bmatrix} x \\ 1 \\ -2 \end{bmatrix}=0$, then the value of x is

(a) $-\frac{1}{2}$ 　　　　(b) $\frac{1}{2}$ 　　　　(c) $\frac{12}{5}$ 　　　　(d) $-\frac{12}{5}$

84. The quadratic expression $17+12x-4x^2$ takes

(a) the least value 6 　　　　　　　　(b) the highest value 26

(c) the highest value 17 　　　　　　　(d) the lowest value 17

85. Three vectors $\overline{A},$ $\overline{B}$ and $\overline{C}$ are given by $\hat{i}+\hat{k},$ $\hat{i}+\hat{j}+\hat{k}$ and $3\hat{i}-2\hat{j}+5\hat{k}$ respectively. Then the vector $\overline{R}$ which satisfies the relation $\overline{R}\times\overline{B}=\overline{C}\times\overline{B}$ and $\overline{R}\cdot\overline{A}=0$ is

(a) $-\hat{i}-6\hat{j}+\hat{k}$ 　　　　　　(b) $\hat{i}+6\hat{j}-\hat{k}$

(c) $2\hat{i}-3\hat{j}+\hat{k}$ 　　　　　　(d) $-\hat{i}+6\hat{j}-\hat{k}$

Rough Work

86. If the magnitude of moment about the point $\hat{j}+\hat{k}$ of a force $\hat{i}+\alpha\hat{j}-\hat{k}$ acting through the point $\hat{i}+\hat{j}$ is $\sqrt{18}$, then the value of α is

(a) 9   (b) 4   (c) ± 2 (d) ± 3

87. The arithmetic mean of n odd natural numbers is

(a) n (b) $\dfrac{n(n+1)}{2}$ (c) $n-1$ (d) n^2

88. A car completes the first half of its journey with a velocity v_1 and the remaining half with velocity v_2. The average velocity of the car for the whole journey is

(a) $\sqrt{v_1 v_2}$ (b) $\dfrac{v_1-v_2}{2}$ (c) $\dfrac{v_1+v_2}{2}$ (d) $\dfrac{2v_1 v_2}{v_1+v_2}$

89. An integer x is chosen at random from the numbers 1 to 28. The probability that $x+\dfrac{192}{x}\le 30$ is

(a) $\dfrac{7}{10}$ (b) $\dfrac{1}{15}$ (c) $\dfrac{2}{28}$ (d) $\dfrac{5}{28}$

90. Let x be a nonzero real number. A determinant is chosen from the set of all determinants of order two with entries x and $!x$ only. The probability that the value of the determinant is nonzero is

(a) $\dfrac{1}{4}$ (b) $\dfrac{1}{2}$ (c) $\dfrac{3}{16}$ (d) $\dfrac{1}{8}$

91. Two candidates A and B are seeking admission in AMRITA UNIVERSITY. The probability that A is selected is 0.5 and the probability that both A and B are selected is atmost 0.25. Then the probability of B getting selected cannot exceed

(a) 0.75   (b) 0.7   (c) 0.8   (d) 0.6

92. The curve satisfying the differential equation $\dfrac{dy}{dx}=\dfrac{y^2-2xy-x^2}{y^2+2xy-x^2}$ and passing through the point $(1,-1)$ is

(a) a circle   (b) a straight line

(c) an ellipse   (d) a parabola

Rough Work

93. The solution of the differential equation $\dfrac{\log dy}{dx} = 9x - 6y + 6$, given $y = 1$ when $x = 0$ is

(a) $3e^{6y} = 2e^{9x-6} + 6e^x$

(b) $3e^{6y} = 2e^{9x+6} - 6e^6$

(c) $3e^{6y} = 2e^{9x+6} + e^6$

(d) $e^{6y} = 2e^{9x-6} + e^{-6}$

94. $\sqrt{2 + \sqrt{2 + \sqrt{2 + 2\cos 8\theta}}}$ is equal to

(a) $2\cos 4\theta$ (b) $2\cos 2\theta$ (c) $2\cos\theta$ (d) $\cos 2\theta$

95. The value of $\lim\limits_{|x| \to \infty} [\cos(\tan^{-1}](\sin(\tan^{-1}x)))$ is equal to

(a) -1 (b) $\sqrt{2}$ (c) $-\dfrac{1}{\sqrt{2}}$ (d) $\dfrac{1}{\sqrt{2}}$

96. If the orthocentre H of a triangle ABC bisects the altitude AD of the triangle ABC, then the value of $\tan B \tan C$ is

(a) 1   (b) 2   (c) 3   (d) 4

97. The remainder got by dividing 2^{804} by 257 is

(a) 16   (b) 15   (c) 17   (d) 14

98. If $\lim\limits_{x \to 0} f(x) = \dfrac{1}{2}$ and $\lim\limits_{x \to 0} g(x) = 4$, then $\lim\limits_{x \to 0} \dfrac{f(x)\cos x}{e^x \sqrt{g(x)}}$ is

(a) 0   (b) -1 (c) 1   (d) 2

99. If $f(x)$ and $g(x)$ are two functions such that $f(2) = 3,$ $g(2) = -4,$ $f'(2) = -\dfrac{1}{2}$ and $g'(2) = -\dfrac{8}{3}$, then the derivative of $\log_e[f(x)g(x) + x]$ at $x = 2$ is

(a) $\dfrac{1}{3}$ (b) $\dfrac{1}{2}$ (c) $-\dfrac{1}{3}$ (d) $-\dfrac{1}{2}$

100. If $p(x)$ is a polynomial of degree three which attains its maximum value 60 at $x = -3$ and minimum value -84 at $x = 3$, then the polynomial is

(a) $\dfrac{x^3}{3} - 9x - 12$

(b) $x^3 - 9x - 12$

(c) $4\left(\dfrac{x^3}{3} - 9x\right) - 12$

(d) $4\left(\dfrac{x^3}{3} - 9x\right) + 12$

Rough Work

101. Part of the domain of the function $f(x) = \sqrt{\dfrac{\cos x - \frac{1}{2}}{6 + 35x - 6x^2}}$ lying in the interval $[-1, \quad 6]$ is

(a) $\left[-\dfrac{1}{6}, \dfrac{\pi}{3}\right] \cup \left[\dfrac{5\pi}{3}, 6\right]$

(b) $\left(-\dfrac{1}{6}, \dfrac{\pi}{3}\right) \cup \left(\dfrac{5\pi}{3}, \quad 6\right)$

(c) $\left(-\dfrac{1}{6}, \quad 6\right)$

(d) $\left(-\dfrac{1}{6}, \quad -6\right)$

102. If the matrix $\begin{bmatrix} 0 & 2\beta & \gamma \\ \alpha & \beta & -\gamma \\ \alpha & -\beta & \gamma \end{bmatrix}$ is orthogonal, then

(a) $\alpha = \pm\dfrac{1}{\sqrt{2}}$

(b) $\beta = \pm\dfrac{1}{\sqrt{6}}$

(c) $\gamma = \pm\dfrac{1}{\sqrt{3}}$

(d) all of these

103. Let $a, \quad b, \quad c$ be positive real numbers. The following system of equations $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1$, $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 2$ and $-\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 3$ has

(a) unique solution

(b) no solution

(c) infinitely many solutions

(d) repeated solutions

104. If the quadratic equation $ax^2 + 2cx + b = 0$ and $ax^2 + 2bx + c = 0$, $(b \neq c)$ have a common root, then $a + 4b + 4c$ is equal to

(a) 0

(b) 1

(c) -1

(d) -2

105. A helicopter is to fly directly from a helipad at the origin in the direction of the point $(1, \quad 1, \quad 1)$ at a speed of $60\ \dfrac{ft}{\sec\square}$. The position of the helicopter after $15\sec\square$ is

(a) $(20\sqrt{3}, \quad 20\sqrt{3}, \quad 20\sqrt{3})$

(b) $(60\sqrt{3}, \quad 60\sqrt{3}, \quad 60\sqrt{3})$

(c) $(300, \quad 300, \quad 300)$

(d) $(300\sqrt{3}, \quad 300\sqrt{3}, \quad 300\sqrt{3})$

106. Let X be the number of times heads occur in n tosses of a fair coin. If $P(X = 4)$, $P(X = 5)$ and $P(X = 6)$ are in A.P., then the least value of n is

(a) 10

(b) 14

(c) 7

(d) 5

Rough Work

107. The solution of the differential equation $\dfrac{dy}{dx} = \dfrac{y\varphi(\varphi'(x) - y^2)}{\varphi(x)}$ is

(a) $y = \dfrac{\varphi(x) + C}{x}$

(b) $y = \dfrac{\varphi(x)}{x + C}$

(c) $y = \varphi(x) + x + C$

(d) $y = \dfrac{\varphi(x)}{x} + C$

108. The solution of the differential equation $\dfrac{dy}{dx} = \sin(x + y) + \cos(x + y)$ is

(a) $\log\left|1 - \tan\left(\dfrac{x+y}{2}\right)\right| = y + C$

(b) $\log\left|1 + \tan\left(\dfrac{x+y}{2}\right)\right| = x + C$

(c) $\log\left|1 + \tan\left(\dfrac{x+y}{2}\right)\right| = y + C$

(d) $\log|1 + \tan(x + y)| = x + C$

109. The equation $\sin^4 x + \cos^4 x + \sin 2x + \beta = 0$ is solvable for

(a) $-\dfrac{5}{2} \le \beta \le \dfrac{1}{2}$

(b) $-3 \le \beta < 1$

(c) $-\dfrac{3}{2} \le \beta \le \dfrac{1}{2}$

(d) $-1 \le \beta \le 1$

110. Given that $x = x(t)$ and $y = y(t)$ satisfy the equations $x + 2x^{\frac{3}{2}} = t^2 + t$ and $y\sqrt{1+t} + 2t\sqrt{y} = 4$, then $\dfrac{dy}{dx}$ at $t = 0$ is

(a) -6
(b) -4
(c) 6
(d) 5

111. Two ships are steaming away from a point 'O' along routes that make an angle of $120°$. Ship A moves at $14\ knots$ and ship B at $21\ knots$. The ships are moving apart at a rate of $'a\ knots'$ when $OA = 5\ nautical\ miles$ and $OB = 3\ nautical\ miles$, where $'a'$ is

(a) 29.5
(b) 28.5
(c) 29
(d) 28

112. If $U_n = \displaystyle\int_0^1 x^n \tan^{-1} x\, dx$, then the value of $(n+1)U_n + (n-1)U_{n-2}$ is

(a) $\dfrac{\pi}{4} - \dfrac{1}{n}$

(b) $\dfrac{\pi}{4} + \dfrac{1}{n}$

(c) $\dfrac{\pi}{2} - \dfrac{1}{n}$

(d) $\dfrac{\pi}{2} + \dfrac{1}{n}$

Rough Work

113. The value of $\int_1^5 2^{\sqrt{x-1}}\, dx$ is

(a) $\dfrac{16}{(log2)^2} - \dfrac{8}{log2}$

(b) $\dfrac{8}{log2} + \dfrac{16}{(log2)^2}$

(c) $\dfrac{8}{log2} - \dfrac{4}{(log2)^2}$

(d) $\dfrac{16}{log2} - \dfrac{8}{(log2)^2}$

114. The pair of tangents drawn from the point $P = (h,\ k)$ to the two circles $x^2 + y^2 + 2x = 0$ and $x^2 + y^2 - 6x = 0$ coincide. Then the point P is

(a) $(-3,\ 2)$　　(b) $(-3,\ 0)$　　(c) $(3,\ 0)$　　(d) $(3, -2)$

115. Two circles pass through $(0,\ \pm a)$ and touch the straight line $x - 2y - 4 = 0$. If the two circles are orthogonal, then the value of a is

(a) $\dfrac{3}{4}$　　(b) $\dfrac{\sqrt{3}}{4}$　　(c) $\dfrac{4}{3}$　　(d) $\dfrac{3}{2}$

116. A force $\bar{F} = 3\hat{i} + \hat{j} - 2\hat{k}$ is applied to a spacecraft with velocity $\bar{v} = \hat{i} - 2\hat{j}$. Then the force F expressed as a vector which is both parallel and orthogonal to $\bar{v}$ is

(a) $\dfrac{1}{5}(14\hat{i} + 7\hat{j} - 2\hat{k})$

(b) $\dfrac{1}{5}(14\hat{i} - 7\hat{j} - 2\hat{k}) + \dfrac{\hat{i} - 2\hat{j}}{\sqrt{5}}$

(c) $\dfrac{14\hat{i}}{5} + \dfrac{7\hat{j}}{5} - 2\hat{k} + \dfrac{\hat{i} - 2\hat{j}}{5}$

(d) $\dfrac{1}{5}(14\hat{i} + 7\hat{j} - 2\hat{k}) + \hat{i} - 2\hat{j}$

117. If $x + 4y - 14 = 0$ is the normal to the curve $y^2 = px^3 + q$ at the point $(2,\ 3)$, then the pair $(p,\ q)$ is

(a) $\{2,\ 7\}$　　(b) $\{-2,\ 7\}$　　(c) $\{3,\ 8\}$　　(d) $\{2, -7\}$

118. The value of the integral $\int \dfrac{log(x+1) - logx}{x(x+1)}\, dx$ is

(a) $C - \dfrac{1}{2}\left(log\left(1 + \dfrac{1}{x}\right)^2\right)$

(b) $log\left(\dfrac{x+1}{x}\right) + C$

(c) $-\dfrac{1}{2}\left(log\left(x - \dfrac{1}{x}\right)\right)^2 + C$

(d) $2\,log\left(x + \dfrac{1}{x}\right) + C$

Rough Work

119. If $\int \frac{x^2+2}{(x^2+1)(x^2+4)} dx = p \tan^{-1}\left(\frac{qx}{r+x^2}\right) + C$, then the values of p , q and r are respectively

(a) $\left\{\frac{1}{3}, -3, -2\right\}$

(b) $\left\{-\frac{1}{3}, 3, 2\right\}$

(c) $\left\{-3, -\frac{2,1}{3}\right\}$

(d) $\left\{\frac{1}{3}, \quad 3, \quad 2\right\}$

120. The area enclosed between the two parabolas $y = 7 - 2x^2$ and $y = x^2 + 4$ is

(a) 3   (b) 4   (c) 2   (d) 5

Rough Work

OMR ANSWER SHEET

1. Use the OMR answer sheet carefully; no spare sheet will be issued under any circumstance.
2. Do not fold or make any stray mark on the OMR sheet.
3. Use HB Pencil or Blue / Black ball point pen for shading the bubbles and black ball pen for writing.
4. In the OMR answer sheet, make the following entries
   a. Write the Registration number, Question Booklet Number and Question Booklet Version code.
   b. Fill the ovals corresponding to the Registration Number, Question Booklet Number and Question Booklet Version Code.
   c. Write your Name and Signature.
5. Rough work should not be done on the answer sheet.

ANSWERING AND EVALUATION

6. For each question, four answers are suggested of which only one is correct / most   appropriate. Mark the correct / most appropriate answer by darkening the corresponding bubble using HB pencil or Blue / Black ball point pen.
7. In case the candidate wishes to change the choice already shaded using HB pencil, he/she may erase the marking completely and thereafter shade the alternative bubble.
8. If more than one bubble is darkened against a question, it will be treated as an incorrect answer.
9. For each correct answer, three marks will be awarded.
10. **For each incorrect answer, one mark will be deducted from the total score.**
11. If any smudge is left on the OMR sheet, evaluation will become imperfect.