46. The Nucleus

Short Answer

1. Question

If neutrons exert only attractive forces, why don't we have a nucleus containing neutrons alone?

Answer

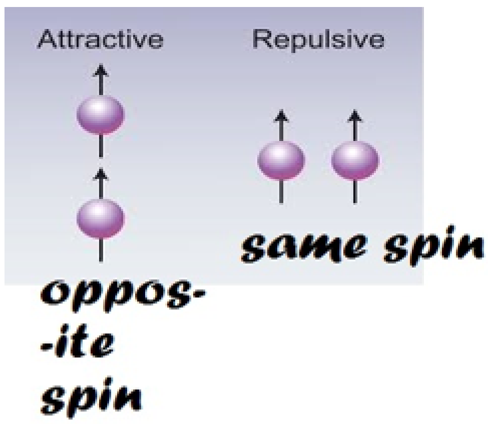
Neutrons exert attractive forces, but are neutral in nature. So there will be no charge inside the nucleus and hence electrons will not face any attraction. Thus the atom in overall will become unstable.

2. Question

Consider two pairs of neutrons. In each pair, the separation between the neutrons is the same. Can the force between the neutrons have different magnitudes for the two pairs?

Answer

The answer can be yes as well as no. The fact is that if the neutrons have the same spin, then the force will be of same magnitude (repulsive) and if they have opposite spin, then force will be of different magnitude(attractive). Such forces are often referred to as tensor forces.



3. Question

A molecule of hydrogen contains two protons and two electrons. The nuclear force between these two protons is always neglected while discussing the behavior of a hydrogen molecule. Why?

Answer

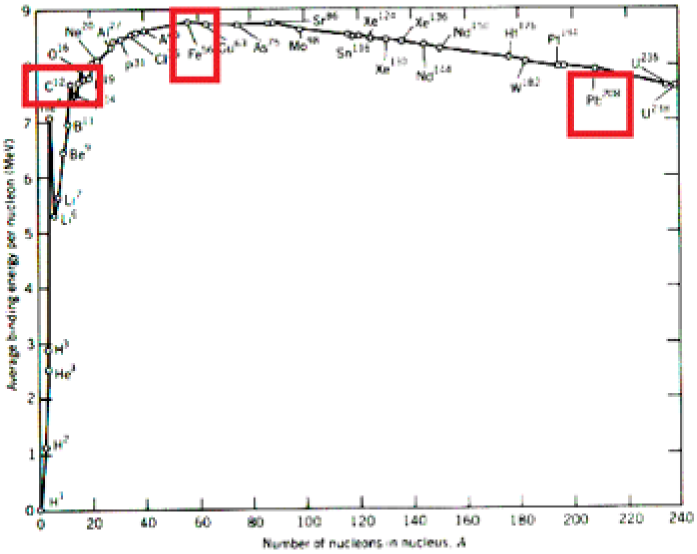
Generally, when we talk about nuclear forces, we restrict our measurement to femtometer(fm)which is a small unit of measurement. But when we discuss of the forces between molecules, we make our measurement in Å which is quite larger than fm. Thus we neglect the nuclear forces between the protons in a H-molecule.

4. Question

Is it easier to take out a nucleon (a)from carbon or from iron (b)from iron or from lead?

Answer

It will be easier to take out nucleon from carbon and lead as iron is most stable among them because it has the highest binding energy per nucleon and thus it is difficult to remove nucleon from the nucleus of iron.

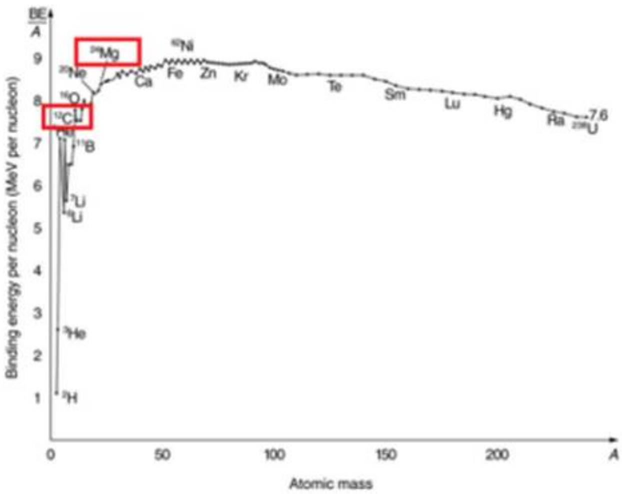


5. Question

Suppose we have 12 protons and 12 neutrons. We can assemble them to form either a ^{24}Mg nucleus or two ^{12}C nuclei. In which of the two cases more energy will be liberated?

Answer

More energy will be liberated in case of ^{24}Mg because it has more binding energy per nucleon as shown in the graph as compared to ^{12}C nuclei.



6. Question

What is the difference between cathode rays and beta rays? When the two are travelling in space, can you make out which is the cathode ray and which is the beta ray?

Answer

Cathode Rays	Beta Rays
1. They contain electron	1. They also consist of electron
2. They get created when high voltage is applied across the electrodes	2. They get created with nuclear fission of radioactive element
3. The electrons in cathode ray get generated through metals(as they conduct electricity)	3. The electrons in beta ray gets generated when neutron breaks to give electron and proton (during nuclear fission)

We will not be able to distinguish between them as they are not visible to human eye because both of the rays consists of electrons.

7. Question

If the nucleons of a nucleus are separated from each other, the mass is increased. Where does this mass come from?

Answer

This appears to be because of mass defect which is commonly known as Einstein's mass defect theory whose proof is yet to be done.

8. Question

In beta decay, an electron (or a positron) is emitted by a nucleus. Does the remaining atom gets oppositely charged?

Answer

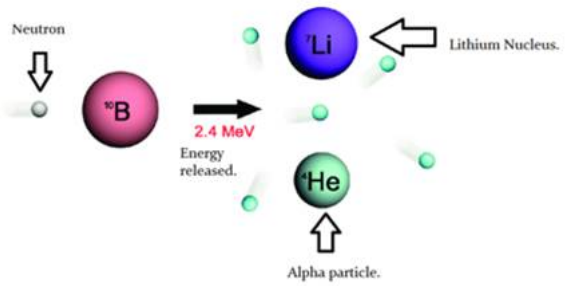
The answer is yes. This is because in case of beta decay an electron comes out from the atom and thus the atom gets oppositely charged as it becomes an ion.

9. Question

When a boron nucleus (^{10}B) is bombarded by a neutron, an α-particle is emitted. Which nucleus will be formed as a result?

Answer

- When boron having atomic no.5 and atomic mass 10 is bombarded with neutron, the atomic no. decreases by 2 i.e.5-2=3 and the atomic mass decreases by 2 times of atomic no. i.e.4. Thus Lithium with atomic no.3 and mass number 6.



10. Question

Does a nucleus loss mass when it suffers gamma decay?

Answer

The answer is no. This is because in a gamma decay, neither the proton number nor the neutron number changes. Only the quantum numbers of nucleons changes.

11. Question

Typically, in a fission reaction, the nucleus split into two middle-weight nuclei of unequal masses. Which of the two (heavier or lighter) has greater kinetic energy? Which one has greater linear momentum?

Answer

-The lighter part will have more kinetic energy.

There can be 2 cases, if the nucleus is at rest, then the two parts will have same linear momentum. If the nucleus is not at rest, then the heavier one will have more linear momentum (as kinetic energy is inversely proportional to momentum).

12. Question

If three helium nuclei combine to form a carbon nucleus, energy is liberated. Why can't helium nuclei combine on their own and minimize the energy?

Answer

-Since the initial separation between helium nuclei are large (in terms of Å), thus nuclear reaction cannot start on their own.

Objective I

1. Question

The mass of a neutral carbon atom in ground state is:

A. exact 12u.

B. less than 12 u.

C. more than 12 u.

D. depends on the form of graphite

such as graphite or charcoal.

Answer

This is because the unit which is widely used in describing mass in nuclear as well as atomic physics is unified atomic mass unit(amu).

This amu is equal to $1/12^{th}$ of the mass of a carbon-12 isotope. Thus for a carbon atom in the ground state, the mass will be equal to 12amu.

Option(b) is not appropriate as because the mass will not be less than 12u (as it is exactly equal to 12 u).

Option(c) is also not appropriate as the mass cannot be more than 12u (as it is exactly equal to 12 u).

Option(d) is not appropriate as the definition describes that the mass is equal to $1/12^{th}$ of carbon-12 isotope. So, it will not depend upon graphite or charcoal.

2. Question

The mass number of a nucleus is equal to

A. the number of neutrons in the nucleus.

B. the number of protons in the nucleus.

C. the number of nucleons in the nucleus.

D. none of them.

Answer

This is because in a nucleus, the mass number is the sum of the total numbers of protons and neutrons together.

Thus (c) will be the correct answer as number of nucleons refers to the total number of protons and neutrons.

3. Question

As compared to ^{12}C atom, ^{14}C atom has,

A. two extra protons and two extra electrons

B. two extra protons but no extra electron

C. two extra neutrons and no extra electron

D. two extra neutrons and two extra electrons

Answer

^{12}C and ^{14}C are the isotopes of carbon. An isotope of an element has the same atomic number but the mass number is different.

The number 12 and 14 in the carbon atom refers to the mass number i.e. the total number of neutrons and protons.

Thus the mass number will increase when the number of neutrons increases. So, the answer is (c).

Option (a) is incorrect as increase in the number of electrons does not change the mass number.

Option (b) is also incorrect. If the number of protons increases but not the number of electrons, then the atomic number of the element will change.

Option (d) is not correct as increasing the number of electrons does not change the mass number.

4. Question

The mass number of a nucleus is

A. always less than its atomic number

B. always more than its atomic number

C. equal to its atomic number.

D. sometimes more than and sometimes equal to its atomic number.

Answer

The fact is that the atomic number is the number of protons and electrons in an atom and mass number is the number of neutrons and electrons, so it can be sometimes more or sometimes less than the atomic number. For example - Hydrogen atom has 1 proton and 1 electron and thus has atomic and mass number as equal.

Option(a) is not correct as it can’t be less than the atomic number because atomic number is the number of protons and mass number is the sum of protons and neutrons.

Option(b) is not appropriate as it can't be always greater than its atomic number as because sometimes the number of the number of protons is less as compared to the electrons that is in case of anion.

Option (c) is not appropriate as mass number is not always equal to its atomic number.

5. Question

The graph of $\ln(R/R_0)$ vs ln A (R=radius of nucleus and A=its mass number) is

A. a straight line

B. a parabola

C. an ellipse

D. none

Answer

As we know that the radius of a nucleus is

$$R = R_0 A^{1/3}$$

Applying log on both the sides, we have,

$$Ln(R) = ln(R_0) \times 1/3\ ln(A)$$

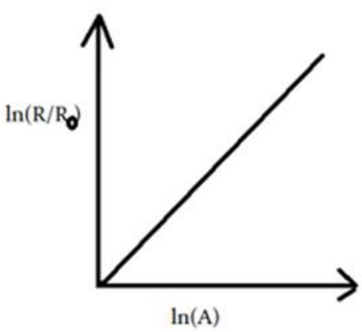
Or $\ln\left(\frac{R}{R_0}\right) = \frac{1}{3}\ln(A)$

Thus the above equation gives us an equation of a straight line

$$y = mx + c$$

Here, $y=\ln(R/R_0)$ and $x=\ln(A)$.

Since in the above equation, c=0.Therefore, the graph obtained will be a straight line passing through the origin.



Thus (a) is the correct answer.

(b) is not the correct answer as the equation of a parabola is

$y = 4ax^2$

(c) is not correct as the equation of ellipse is

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

6. Question

Let Fpp, Fpn and Fnn denote the magnitude of the nuclear force by a proton on a proton, by a proton on a neutron and by a neutron on a neutron respectively. When the separation is 1fm,

A. Fpp>Fpn=Fnn

B. Fpp=Fpn=Fnn

C. Fpp>Fpn>Fnn

D. Fpp<Fpn=Fnn.

Answer

As the distance of separation is equal, so the forces that will act between the particles will be same as because

Mass of neutron=Mass of proton=$1.67 \times 10^{-27} kg$

Thus, the forces can be represented as,

Fpp=Fnn=Fpn=$\frac{kQq}{r}$

$$or \ \frac{k \times 1.67 \times 10^{-19} \times 1.67 \times 10^{-19}}{1fm}$$

Therefore, as the distance of separation is equal, thus they will have same magnitude of forces.

So, (b) is the correct answer.

7. Question

Let Fpp, Fpn and Fnn denote the magnitudes of the net force by a proton on a proton, proton on a neutron and neutron on a neutron. Neglect gravitational force. When the separation is 1fm,

A. Fpp>Fpn=Fnn

B. Fpp=Fpn=Fnn

C. Fpp>Fpn>Fnn

D. Fpp<Fpn=Fnn.

Answer

Nuclear forces are much stronger as compared to gravitational and electromagnetic forces.

As neutrons does not have any charge, thus the force between 2 neutron and between a proton and a neutron will be zero as

$$Fpn = Fnn = \frac{(kqQ)}{r} = 0\ (as\ charge\ of\ neutron\ is\ zero)$$

Also, the force between 2 protons is columbic in nature. This coulomb force is weaker as compared to the nuclear forces between proton-proton and neutron-proton. Therefore, option (d) is the correct option.

Option (a) is not correct as the force between proton-proton can't be greater than those of neutron-neutron and proton-neutron.

Option (b) is not correct as the forces can't be same because coulomb force and nuclear force are not equal in magnitude.

Option (c) also can't be correct as because force between proton-neutron and neutron-neutron are all nuclear force.

8. Question

Two protons are kept at a separation of 10nm.Let Fn and Fe be the nuclear force and electromagnetic force between them.

A. Fe=Fn

B. Fe>>Fn

C. Fe<<Fn

D. Fe and Fn differ only slightly.

Answer

The correct answer will be (b).This is because electromagnetic force is $10^{36} stronger$.Whereas nuclear force is $10^{25} stronger.$

Option (a) is not correct as nuclear forces and electromagnetic forces can't be equal because electromagnetic force is much stronger than nuclear force.

Option (c) is not correct as nuclear force is not stronger than electromagnetic force.

Option (d) is also not correct as they differ by a large margin.

9. Question

As the mass number A increases, the binding energy per nucleon in a nucleus

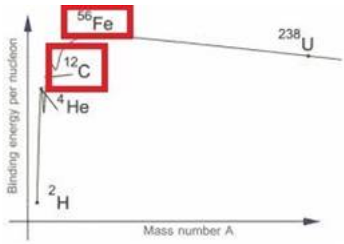
A. increases

B. decreases

C. remains the same

D. varies in a way that depends on the actual value of A

Answer



Clearly from graph iron(Fe)has more binding energy per nucleon than carbon

Thus the value of binding energy will increase as the mass number increases.

10. Question

Which of the following the wrong description of the binding energy of the nucleus?

A. It is the energy required to break a nucleus into its constituent nucleons.

B. It is the energy made available when free nucleons combine to form a nucleus

C. It is the sum of the rest mass energies of its nucleons minus the rest mass energy of the nucleus.

D. It is the sum of kinetic energies of all the nucleons in the nucleus.

Answer

The correct answer is (d). All the other options are correct about binding energy per nucleon.

This is because inside the nucleus, the nucleons are in rest. So, there is no question of kinetic energy for those particles at rest.

11. Question

In one average-life,

A. half the active nuclei decay.

B. less than half the active nuclei decay.

C. more than half the active nuclei decay.

D. all the nuclei decay.

Answer

The correct answer is (c). This is because, the formula for average life calculation is,

$$t_{av} = \frac{t\left(\frac{1}{2}\right)}{0.693}$$

So the decay will be more than half as dividing the half-life with 0.693 will give the result to be more than half of the active nuclei will decay.

12. Question

In a radioactive decay, neither the atomic number nor the mass number changes. Which of the following particles is emitted in the decay?

A. Proton

B. Neutron

C. Electron

D. Photon

Answer

Atomic number is the number of protons while atomic mass number is the sum of protons and neutrons. The particles emitted will not be proton, electron or neutron since there is no change in the atomic number and mass number. Hence, the emitted particle must be photon.

13. Question

During a negative beta decay,

A. an atomic electron is ejected

B. an electron which is already present within the nucleus is ejected.

C. a neutron in the nucleus decays emitting an electron

D. a proton in the nucleus decays emitting an electron.

Answer

Negative beta decay is a process in which an unstable nucleus, formed with more number of neutrons than needed for stability, tries to go towards stability by converting a neutron to a proton. In this process, the unstable nucleus emits an electron and an antineutrino i.e. $n \rightarrow p + e + antineutrino$. Thus, we can say that a neutron in the nucleus decays, emitting an electron.

14. Question

A freshly prepared radioactive source of half-life 2h emits radiation of intensity which is 64 times the permissible safe level. The minimum time after which it would be possible to work safely with this source is

A. 6 h

B. 12 h

C. 24 h

D. 128 h

Answer

The intensity of radiation emitted by radioactive source also decreases with time. Half-life of the given material is 2 hours. Let the radiation starts at t=0. Initial intensity of radiation is 64 times the safe intensity level.

At t=2 hour, the intensity of radiation will be half (due to half-life) which is now 32 times the safe intensity level.

At t=4 hour, the intensity of radiation will be 16 times the safe level.

Doing this, t=6 h $\Longrightarrow$ 8 times; t=8 h $\Longrightarrow$ 4 times; t=10 h $\Longrightarrow$ 2 times and t=12 h $\Longrightarrow$ 1 times the safe intensity level. In other words, $2^6 = 64$ and thus the time will be 6*2 hour= 12 hour. Therefore, the minimum time is 12 hours after which the intensity of radiation is equal to the permissible safe level and we can work safely with the material.

15. Question

The decay constant of a radioactive sample is λ. The half-life and the average-life of the sample are respectively.

A. 1\λ and (ln 2\λ)

B. (ln 2\λ) and 1\λ

C. λ (ln 2) and 1\λ

D. λ/ (ln 2) and 1\λ

Answer

The half-life of a radioactive material is given by

$$t_{1/2} = \frac{\ln 2}{\lambda}$$

The average life of a radioactive material is given by

$$t_{av} = \frac{1}{\lambda}$$

16. Question

An α-particle is bombarded on ^{14}N. As a result, a ^{17}O nucleus is formed and a particle is emitted. This particle is a

A. neutron

B. proton

C. electron

D. positron

Answer

An α-particle is ^4_2He. ^4_2He is bombarded on $^{14}_7N$ and thus $^{17}_8O$ is formed with an emission of a particle given by following equation

$$^{14}_7N + ^4_2He \rightarrow ^{17}_8O + ^1_1x$$

The particle emitted is ^1_1x which is equal to a proton since $Z = 1$.

17. Question

Ten grams of ^{57}Co kept in an open container beta-decays with a half-life of 270 days. The weight of the material inside the container after 540 days will be very nearly.

A. 10g

B. 5g

C. 2.5g

D. 1.25g

Answer

A neutron is converted to a proton with creation of an electron and an antineutrino during beta-decay. The rest mass of antineutrino is approximately zero (though unknown in physics till now) while the rest mass of an electron is $9.11 * 10^{-31}$kg. Thus the mass of a beta particle is much less when compared to the mass of Co atom. Hence even after 540 days in a container, the weight of the material will be approximately equal to 10g.

18. Question

Free ^{238}U nuclei kept in a train emit alpha particles. When the train is stationary and a uranium nucleus decays, a passenger measures that the separation between the alpha particle and the recoiling nucleus becomes x in time t after the decay. If a decay takes place when the train is moving at a uniform speed v, the distance between the alpha particle and the recoiling nucleus at a time t after the decay, as measured by the passenger will be

A. x + vt

B. x – vt

C. x

D. depends on the direction of the train.

Answer

The separation between the alpha particle and the recoiling nucleus is measured as x in time t when uranium atom is kept in a stationary train. If decay takes place in a moving train, the distance between the alpha particle and the recoiling nucleus is now measured, will remain x in time t. This is because the decay process and the passenger are in same time frame.

19 OBJECTIVE. Question

During a nuclear fission reaction,

A. a heavy nucleus breaks into two fragments by itself

B. a light nucleus bombarded by thermal neutrons breaks up

C. a heavy nucleus bombarded by thermal neutrons breaks up

D. two light nuclei combine to give a heavier nucleus and possible other products.

Answer

The reaction in which the nuclear energy is obtained by breaking a heavy nucleus into two or more light nuclei is called as nuclear fission reaction. Highly fissionable material like $^{236}_{92}U$ are not found in nature. However, the natural Uranium contains 0.7% of $^{235}_{92}U$ which has high probability of absorbing a slow neutron and thus forming$^{236}_{92}U$. Thus, a heavy nucleus $^{235}_{92}U$ is bombarded by thermal neutrons which results in formation of $^{236}_{92}U$. This highly fissionable material now breaks up and nuclear fission reaction occurs.

Objective II

1. Question

As the mass number A increases, which of the following quantities related to a nucleus do not change?

A. Mass

B. Volume

C. Density

D. Binding energy

Answer

The average radius R of a nucleus is given by

$R = R_0 A^{1/3} \cdots (1)$

Where A is the mass number and $R_0 = 1.1 * 10^{-15}$m.

The Volume of the nucleus V is given by (Putting equation (1))

$$V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi {R_0}^3 A \cdots (2)$$

Since, the number of protons and neutrons $(Z + N = A)$ are nearly same, say m, the Mass M is approximately equal to

$$M = mA \cdots (3)$$

Density is given by

$$\rho = \frac{M}{V} = \frac{3m}{4\pi {R_0}^3} \cdots (from\ equation\ (2) and\ (3))$$

The above equation of density says that it is independent of A. So as mass number A increases, density do not change.

2. Question

The heavier nuclei t end to have larger N/Z ratio because

A. a neutron is heavier than a proton

B. a neutron is an unstable particle

C. a neutron does not exert electric repulsion

D. Coulomb forces have longer range compared to the nuclear forces.

Answer

Heavier nuclei having large mass number A has large nuclei radius$\left(R \propto A^{1/3}\right)$. Attractive nuclear force, which has small range, is not effective in heavier nuclei due to the large nuclei radius. Repulsive Coulomb forces between protons will be now effective since these forces have longer range. Repulsive force causes instability of nuclei.

Stability is achieved by having more neutrons than protons in the nuclei. More neutrons (not exerting electric repulsion) in the nuclei or larger N/Z ratio do make the range between the nucleon-pairs comparable to the nuclear forces. Hence, attractive nuclear forces dominate over repulsive coulomb forces and thus stability is achieved. Therefore, Option C and D both are correct.

3. Question

A free neutron decays to a proton but a free proton does not decay to a neutron. This is because

A. neutron is a composite particle made of a proton and an electron whereas proton is a fundamental particle

B. neutron is an uncharged particle whereas proton is a charged particle

C. neutron has larger rest mass than the proton

D. weak forces can operate in a neutron but not in a proton.

Answer

When a free neutron decays to a proton, an electron and an antineutrino are created i.e. $n \rightarrow p + e + antineutrino$. The rest mass of neutron is larger than that of proton and thus the Q-value is positive. If we consider a free proton decays to a neutron, a positron and a neutrino are created i.e. $p \rightarrow n + e^+ + neutrino$. The Q-value is thus negative which is impossible. Also, a lower mass particle cannot be decayed into a large mass particle. Hence, a free proton does not decay to a neutron.

4. Question

Consider a sample of a pure beta-active material.

A. All the beta particles emitted have the same energy.

B. The beta particles originally exist inside the nucleus and are ejected at the time of beta decay.

C. The antineutrino emitted in a beta decay has zero mass and hence zero momentum.

D. The active nucleus changes to one of its isobars after the beta decay.

Answer

Since an electron and an antineutrino are emitted from a pure beta-active material, these particles do not have same energy due to their different masses. The beta particles are ejected when a neutron is converted into a proton and thus we can't say that beta particles were already present in the nucleus. The mass of antineutrino is unknown. Hence, Option A, B and C are not correct. Nuclei with same mass number but different atomic number are called *isobars.* The beta decay process is ${}_Z^AX \rightarrow {}_{Z+1}^{A}Y + e + antineurtino$. It is prevalent that, $A = Z + N$ (Before beta decay) while $A = (Z + 1) + (N - 1) = Z + N$ (after beta decay). Thus we say that the active nucleus changes to one of its isobars after the beta decay.

5. Question

In which of the following decays the element does not change?

A. α-decay

B. β^+-decay

C. β^--decay

D. γ-decay

Answer

α-decay: ${}_{Z}^{A}X \rightarrow {}_{Z-2}^{A-4}Y + {}_{2}^{4}He$ $\Rightarrow$ Element changes.

β^+-decay: ${}_{Z}^{A}X \rightarrow {}_{Z-1}^{\ \ A}Y + e^+ + neutrino$ $\Rightarrow$ Element changes.

β^--decay: ${}_{Z}^{A}X \rightarrow {}_{Z+1}^{\ \ A}Y + e^- + antineutrino$ $\Rightarrow$ Element changes.

γ-decay: This decay is a radioactive process in which the excited nucleus comes down to its ground energy level by emitting photons. The element does not change in this process.

6. Question

In which of the following decays the atomic number decreases?

A. α-decay

B. β^+-decay

C. β^--decay

D. γ-decay

Answer

α-decay: ${}_{Z}^{A}X \rightarrow {}_{Z-2}^{A-4}Y + {}_{2}^{4}He$ $\Rightarrow$ Atomic number decreases by 2.

β^+-decay: ${}_{Z}^{A}X \rightarrow {}_{Z-1}^{\ \ A}Y + e^+ + neutrino$ $\Rightarrow$ Atomic number decreases by 1.

β^--decay: ${}_{Z}^{A}X \rightarrow {}_{Z+1}^{\ \ A}Y + e^- + antineutrino$ $\Rightarrow$ Atomic number increases by 1.

γ-decay: This decay is a radioactive process in which the excited nucleus comes down to its ground energy level by emitting photons. Atomic number does not change.

7. Question

Magnetic field does not cause deflection in

A. α-rays

B. beta-plus rays

C. beta-minus rays

D. gamma rays

Answer

Magnetic field gets deflected when there is charge/current within its surroundings.

α-decay: ${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He \Rightarrow {}^{4}_{2}He$ causes deflection in magnetic field.

β^{+}-decay: ${}^{A}_{Z}X \rightarrow {}_{Z-1}{}^{A}Y + e^{+} + neutrino \Rightarrow e^{+}$ causes deflection in magnetic field.

β^{-}-decay: ${}^{A}_{Z}X \rightarrow {}_{Z+1}{}^{A}Y + e^{-} + antineutrino \Rightarrow e^{-}$ causes deflection in magnetic field.

γ-decay: This decay is a radioactive process in which the excited nucleus comes down to its ground energy level by emitting photons. Photons are not deflected by magnetic field.

8. Question

Which of the following are electromagnetic waves?

A. α-rays

B. Beta-plus rays

C. Beta-minus rays

D. Gamma rays

Answer

Photons constitute electromagnetic waves. Since gamma rays are photons emitted during nuclear transitions, they are electromagnetic waves.

9. Question

Two lithium nuclei in a lithium vapour at room temperature do not combine to form a carbon nucleus because

A. a lithium nucleus is more tightly bound than a carbon nucleus

B. carbon nucleus is an unstable particle

C. it is not energetically favorable

D. Coulomb repulsion does not allow the nuclei to come very close.

Answer

In a lithium vapor at room temperature, the distance between two lithium nuclei is larger when compared to the short-range attractive nuclear forces. Thus, repulsive coulomb forces will be effective and this does not allow the two nuclei to come very close to form a carbon nucleus. If we want to combine two lithium nuclei to form a carbon nucleus, we need a temperature of the order of 10^9 K.

10. Question

For nuclei with A > 100,

A. the binding energy of the nucleus decreases on an average as A increases

B. the binding energy per nucleon decreases on an average as A increases

C. if the nucleus breaks into two roughly equal parts, energy is released

D. if two nuclei fuse to form a bigger nucleus, energy is released.

Answer

For A=50 to A=80, the binding energy per nucleon increases on an average. For A>80, the binding energy per nucleon decreases on an average. Therefore, Option B is correct. For heavy nuclei with A>100, the unstable nucleus can break into two roughly equal parts with release of energy to attain stability. Therefore, option C is correct. For heavy nuclei with A>100, it is impossible to combine two nuclei to form a bigger nucleus. Lighter nuclei with A<100 can be combined to form a bigger nucleus with release of energy. Therefore, option D is not correct for A>100.

Exercises

1. Question

Assume that the mass of a nucleus is approximately given by $M = Am_p$ where A is the mass number. Estimate the density of matter in kg m^{-3} inside a nucleus. What is the specific gravity of nuclear matter?

Answer

Given mass of nucleus(M) = Am_p

We know that,

$mp = 1.007276\ u$ (mass of proton)

where µ is the atomic mass unit ,$u = 1.6605402 \times 10^{-27}\ kg$

Radius of nucleus(R) $= R_0\ A^{\frac{1}{3}}$

$= 1.1 \times 10^{-15}\ A^{\frac{1}{3}}\ m$

Volume(V) $= \frac{4}{3}\pi R^3 = \frac{4}{3}\pi \left(R_0 A^{\frac{1}{3}}\right)^3$

$= \frac{4}{3}\pi R_0^3 A$

Density(D) $= \frac{Mass(M)}{Volume(V)}$

$$= \frac{A \times 1.007276 \times 1.6605402 \times 10^{-27}}{\frac{4}{3} \times \pi \times R_0^3 \times A}$$

= 3.0000688×10^{17} kg/m^3

Specific gravity $= \frac{Density(in\,SI)}{1000}$

= 3.0000688×10^{14}.

Hence, the density of matter in kg m^{-3} inside a nucleus is 3.0000688×10^{17} kg/m^3 and its specific gravity is 3.0000688×10^{14}.

2. Question

A neutron star has a density equal to that of the nuclear matter. Assuming the star to be spherical, find the radius of a neutron star whose mass is 4.0×10^{30} kg (twice the mass of the sun).

Answer

Given mass of star(M) = 4×10^{30} kg

We know,

Density of nuclear matter(D) = 2.3×10^{17} kg/m^3

$$Density = \frac{Mass}{Volume}$$

$$Volume = \frac{Mass}{Density}$$

$$= \frac{4 \times 10^{30}}{2.3 \times 10^{17}} m^3$$

$$= \frac{4}{2.3} \times 10^{13} m^3$$

Again, Volume $= \frac{4}{3}\pi R^3$ [As the star is assumed to be a sphere]

[R = Radius of the neutron star]

So equating the expressions of volume we can say,

$$\frac{4}{3}\pi R^3 = \frac{4}{2.3} \times 10^{13}$$

$$\Rightarrow R^3 = \frac{3 \times 10^{13}}{2.3 \times \pi}$$

$\Rightarrow R = \left(\frac{3\times10^{13}}{2.3\times\pi}\right)^{\frac{1}{3}}$

$\Rightarrow R = 16071.2964\ m$

$\Rightarrow R = 16.07\ km$

Hence radius of the star is 16.07 km

3. Question

Calculate the mass of an α-particle. Its binding energy is 28.2 MeV.

Answer

We know that an alpha particle consists of two protons and two neutrons.

Mass of proton = 1.007276 u

Mass of neutron = 1.008665 u

Where u = 1.6605402 × 10^{-27} kg (Atomic mass unit)

Let the mass of the alpha particle be M

∴ Mass defect is ΔM,

$$\Delta M = (no.of\ proton \times Mass\ of\ proton + no.of\ nuetron \times mass\ of\ neutron) - M$$

$$\Delta M = (2 \times 1.007276\ \mathrm{u} + 2 \times 1.008665\ \mathrm{u}) - M$$

Given, Binding energy = 28.2 MeV

Binding energy is also equal to ΔMc^2

So,

$$\Delta Mc^2 = 28.2$$

$\Delta M = \frac{28.2}{c^2}$ [Note $c^2 = 931.5\ MeV/u$]

$$\Delta M = 0.030273u$$

$$(2 \times 1.007276u + 2 \times 1.008665u) - M = 0.030273u$$

$$M = [(2 \times 1.007276 + 2 \times 1.008665) - 0.030273\ ]u$$

$$M = 4.0016\ u$$

Hence, the mass of an alpha particle is 4.0016u

4. Question

How much energy is released in the following reaction:

$^{7}Li + p \rightarrow \alpha + \alpha$.

Atomic mass of ^{7}Li = 7.0160 u and that of ^{4}He = 4.0026 u.

Answer

Given,

Atomic mass of ^{7}Li = 7.0160 u

Mass of proton = 1.007276 u

Atomic mass of ^{4}He = Mass of α particle = 4.0026 u

$\therefore Mass\ defect(\Delta M) = (mass\ of\ reactants) - (mass\ of\ products)$

$Mass\ defect(\Delta M) = [\,(7.0160 + 1.007276)u - 2 \times (4.0026\ u)]$

So, ΔM = 0.018076u

Binding Energy $= \Delta Mc^2$ [Where $c^2 = 931.5\ MeV/u$]

$= 0.018076 \times 931.5\ MeV$

$= 16.83\ MeV$

Hence, the binding energy for the reaction is 16.83 MeV

5. Question

Find the binding energy per nucleon of $^{197}_{79}Au$ if its atomic mass is 196.96 u.

Answer

$^{A}_{Z}X$, here X = elementA = Mass Number[No. of protons + neutrons]

Z = Atomic Number [No. of protons]

Therefore, Number of neutrons(N)= A – Z

Now coming to the problem,

$^{197}_{79}Au$ means that A = 197 and Z = 79

N = A – Z = 197 – 79 = 118

Binding energy $= [(Z_{mp} + N_{mn}) - M]c^2$, m_p = mass of proton

m_n= mass of neutron

M = Atomic Mass

c^2 = (Speed of light)^2

= 931.5 MeV/u

Binding energy

$$= [(79 \times 1.007276 + 118 \times 1.008665) - 196.96] \times 931.5\ MeV$$

$$= 1525.12\ MeV$$

Number of nucleons = No. of protons + neutrons = A = 197

Binding energy per nucleon $= \frac{total\ binding\ energy}{no\ of\ nucleons} = \frac{1525.12}{197} = 7.741$

Hence, the binding energy per nucleon = 7.741

6. Question

(a) Calculate the energy released if ^{238}U emits an α-particle.

(b) Calculate the energy to be supplied to ^{238}U if two protons and two neutrons are to be emitted one by one. The atomic masses of ^{238}U, ^{234}Th and ^{4}He are 238.0508 u, 234.04363 u and 4.00260 u respectively.

Answer

a) $U^{238} \rightarrow Th^{234} + He_2^4$ [Reaction for emitting α-particle]

$_2He^4$ = alpha particle

$$Energy\ released\ = [mass\ of\ reactant - mass\ of\ products] \times c^2$$

$$= [M_u - (M_\alpha + M_{Th})] \times c^2\ ,$$

M_u = mass of Uranium

M_α = mass of α-particle

M_{th} = mass of Thorium

c^2= (Speed of light)^2 = 931.5 MeV/u

$$= [238.0508 - (234.04363 + 4.00260)]u \times 931.5\frac{MeV}{u}$$

$$E = 4.2569\ MeV$$

The energy released is 4.2569 MeV

b) $U^{238} \rightarrow Th^{234} + 2p^1 + 2n^1$

$$Energy\ released\ = [mass\ of\ reactant - mass\ of\ products] \times c^2$$

$= [M_u - \{2 \times (M_p + M_n) + M_{Th}\}] \times c^2]$

M_p = mass of proton=$1.007276u$

M_n = mass of neutron=$1.008665u$

$$= [238.0508 - 2 \times (1.007276 + 1.008665 + 234.04363)]u \times 931.5\frac{MeV}{u}$$

$= 23.019\ MeV$

The energy released is 23.019 MeV

7. Question

Find the energy liberated in the reaction

$^{223}Ra \rightarrow {}^{209}Pb + {}^{14}C$.

The atomic masses needed are as follows.

^{223}Ra ^{209}Pb ^{14}C

223.018 u 208.981 u 14.003 u

Answer

$Ra^{223} \rightarrow Pb^{209} + C^{14}$

$Energy\ released\ = [mass\ of\ reactant - mass\ of\ products] \times c^2$

$Energy\ released\ = [M_{Ra} - (M_{Pb} + M_C)\ \}] \times c^2]$

M_{Pb} = mass of Pb^{209}=208.981 u

M_C = mass of C^{14} =14.003 u

M_{Ra} = mass of Ra^{223}=223.018 u

$= [223.018 - (208.981 + 14.003)] \times 931.5\ MeV$

$= 31.671\ MeV$

The energy released is 31.671 MeV

8. Question

Show that the minimum energy needed to separate a proton from a nucleus with Z protons and N neutrons is

$\Delta E = (M_{Z-1,\ N} + M_H - M_{Z,\ N})\ c^2$

Where $M_{Z,N}$ = mass of an atom with Z protons and N neutrons in the nucleus and M_H = mass of a hydrogen atom. This energy is known as proton-separation energy.

Answer

$X_{Z,N} \rightarrow X_{Z-1,N} + H_1$, X = element under consideration

[Note: Hydrogen has no neutrons in the nucleus]

Now from the above equation we can easily write the energy equation:

$$\Delta E = (mass\ of\ products - mass\ of\ reactans)c^2$$

Hence

$$\Delta E = (M_{Z-1,N} + M_H - M_{Z,N})c^2$$

9. Question

Calculate the minimum energy needed to separate a neutron from a nucleus with Z protons and N neutrons in terms of the masses $M_{Z,N}$, $M_{Z,N-1}$ and the mass of the neutron.

Answer

$X_{Z,N} \rightarrow X_{Z,N-1} + n_0^1$

Energy released = Mass difference × c^2

c^2 = (speed of light)^2 = 931.5 MeV/u

$$\Delta E = (mass\ of\ products - mass\ of\ reactans)c^2$$

$$= (M_{Z,N-1} + M_N - M_{Z,N}) \times c^2$$

$$\therefore \Delta E = (M_{Z,N-1} + M_N - M_{Z,N}) \times c^2$$

10. Question

^{32}P beta-decays to ^{32}S. Find the sum of the energy of the antineutrino and the kinetic energy of the β-particle. Neglect the recoil of the daughter nucleus. Atomic mass of ^{32}P = 31.974 u and that of ^{32}S = 31.972 u.

Answer

$P^{32} \rightarrow S^{32} + \bar{v}_0^0 + \beta_1^0$

Sum of energy of antineutrino and β-particle is must be equal to the energy difference between the P^{32} and S^{32} nuclei

$P^{32} - S^{32} \rightarrow + \bar{v}_0^0 + \beta_1^0$

Energy = $(mass\ of\ P^{32} - mass\ of\ S^{32}) \times c^2$

where c^2= (Speed of light)^2 = 931.5 MeV/u

$$= (31.974\text{ u} - 31.972\text{ u}) \times 931.5\frac{\text{MeV}}{\text{u}}$$

$$= 0.002 \times 931.5\ MeV$$

$$= 1.863\ MeV$$

Hence, the of sum of energy of antineutrino and β-particle is 1.863 MeV

11. Question

A free neutron beta-decays to a proton with a half-life of 14 minutes.

(a) What is the decay constant?

(b) Find the energy liberated in the process.

Answer

a) Given, half-life = 14 minutes = 840 seconds

We know, half life $= \frac{\ln 2}{\lambda}$, where λ = decay constant

$\lambda = \frac{\ln 2}{840}$ = $8.25\times10^{-4}\ s^{-1}$

b) $n_0^1 \rightarrow p_1^1 + \beta_{-1}^0$

[equation of neutron undergoing β decay]

$$Energy\ released\ = [mass\ of\ reactant - mass\ of\ products] \times c^2$$

$$\text{Energy} = \left[\ M_n - \left(M_p + M_\beta\right)\right] \times c^2$$

M_n = mass of Neutron

M_p = mass of proton

M_β = mass of β-particle

c^2= (Speed of light)^2 = 931.5 MeV/u

$$= [1.008665 - (1.007276\ +\ 0.0005486)] \times 931.5\ MeV$$

$$= 0.78283\ MeV$$

= 782.83 KeV

12. Question

Complete the following decay schemes.

(a) $^{226}Ra \rightarrow \alpha +$

(b) $^{19}O \rightarrow {}^{19}_{9}F +$

(c) ${}^{25}_{13}Al \rightarrow {}^{25}_{12}Mg +$

Answer

a) As one alpha particle is produced so the mass number will decrease by 4 and the atomic number by 2,

Resultant reaction:

$$Ra^{226} \rightarrow \alpha_2^4 + Rn^{222}$$

b) As fluorine is produced so the atomic number increases by 1, which suggests β-decay and usually β decay occurs with a loss of an antineutrino

$$O_8^{19} \rightarrow F_9^{19} + \beta_{-1}^{0} + \bar{v}_0^0$$

c) A similar reaction as the previous one, atomic number decreases by one so β^+ emission occurs

$$Al_{13}^{25} \rightarrow Mg_{12}^{25} + \beta_1^0 + \bar{v}_0^0$$

13. Question

In the decay $^{64}Cu \rightarrow {}^{64}Ni + e^+ + v$, the maximum kinetic energy carried by the positron is found to be 0.650 MeV.

(a) What is the energy of the neutrino which was emitted together with a positron of kinetic energy 0.150 MeV?

(b) What is the momentum of this neutrino in kg ms^{-1}? Use the formula applicable to a photon.

Answer

$$Cu^{64} \rightarrow Ni^{61} + e^+ + v$$

a) Maximum energy of positron refers to zero K.E of the neutrino

So when the energy of positron is 0.150 MeV as energy is conserved,

Energy of neutrino $= Max\ K.E\ of\ positron - Present\ energy\ of\ positron$

$= 0.650 - 0.150\ MeV$

$= 0.500\ MeV$

b) For a photon,

$momentum = \frac{Energy}{c}$, where c = speed of light

Energy of neutrino in SI = $0.500 \times 10^6 \times 1.6 \times 10^{-19}$ J

Momentum = $\frac{0.500 \times 10^6 \times 1.6 \times 10^{-19}}{3 \times 10^8}$ $kgms^{-1}$

= 2.67×10^{-22} $kgms^{-1}$

14. Question

Potassium -40 can decay in three modes. It can decay by β^--emission, β^+-emission or electron capture.

(a) Write the equations showing the end products.

(b) Find the Q-value in each of the three cases. Atomic masses of ${}^{40}_{18}Ar$, ${}^{40}_{19}K$ and ${}^{40}_{20}Ca$ are 39.9624 u, 39.9640 u and 39.9626 u respectively.

Answer

a) Equations:

1) $K^{40}_{19} \rightarrow Ca^{40}_{20} + e^{0}_{-1} + energy$ [β^- emission]

2) $K^{40}_{19} \rightarrow Ar^{40}_{18} + e^{0}_{+1} + energy$ [β^+ emission]

3) $K^{40}_{19} + e^{0}_{-1} \rightarrow Ar^{40}_{18} + energy$ [electron capture]

b)

$$Q\ value\ =\ [Mass\ of\ reactants - Mass\ of\ products]c^2,$$

where c^2 = 931.5 MeV/u

case 1:

$$Q\ =\ [39.9640 - 39.9626] \times\ 931.5\ MeV$$

$$=\ 1.3041\ MeV$$

case 2:

$$Q\ =\ [39.9640 - (39.9624 + 2 \times 0.005486)] \times 931.5\ MeV$$

$$=\ 0.4683\ MeV$$

Case 3:

$$Q\ = [39.9640 - 39.9624] \times 931.5\ MeV$$

$$=\ 1.490\ MeV$$

Note: For energy calculations we use the masses of the nuclei of the reactions but as we are given atomic masses we add and subtract appropriate number of electrons to get the above expressions. For more details, one should look through the derivation of Q value for the three cases.

15. Question

Lithium (Z = 3) has two stable isotopes ^{6}Li and ^{7}Li. When neutrons are bombarded on lithium sample, electrons and α-particles are ejected. Write down the nuclear processes taking place.

Answer

Considering the data given in the question only 2 isotopes are stable

$Li_3^6 + n \rightarrow Li_3^7$ $[Stable]$

$Li_3^7 + n \rightarrow Li_3^8$ $[Unstable]$

So Li_3^8 ejects and electron to convert to more stable Be and which in turn ejects two α particles

$Li_3^8 \rightarrow Be_4^8 + e^- + energy$

$Be_4^8 \rightarrow 2He_2^4$

16. Question

The masses of ^{11}C and ^{11}B are respectively 11.0114 u and 11.0093 u. Find the maximum energy a positron can have in the β+-decay of ^{11}C to ^{11}B

Answer

$C_6^{11} \rightarrow B_5^{11} + \beta_1^0$

The maximum energy for the positron will be equal to the energy due to the mass defect (ΔM)

ΔM = [11.0114-11.0093] ×931 MeV

= 1.955 MeV

17. Question

^{228}Th emits an alpha particle to reduce to ^{224}Ra. Calculate the kinetic energy of the alpha particle emitted in the following decay:

^{228}Th → ^{224}Ra* + α

^{224}Ra* → ^{224}Ra + γ (217 keV).

Atomic mass of ^{228}Th is 228.028726u, that of ^{224}Ra is 224.020196 u and that of $_2^4He$ is 4.00260 u.

Answer

Energy equation for the reaction $^{224}Ra^* \rightarrow {}^{224}Ra + \gamma$ will be

$$(M_{Ra^*} - M_{Ra}) \times c^2 = 0.217 \text{ Mev}$$

$$(M_{Ra^*} - 224.020196 \text{ u}) \times 931 \frac{\text{MeV}}{\text{u}} = 0.217\ MeV$$

$$M_{Ra^*} - 224.020196 \text{ u} = 0.00023333u$$

$$M_{Ra^*} = 224.02042u$$

Now for the first equation:

$^{228}Th \rightarrow {}^{224}Ra^* + \alpha$

K.E of α particle $= (Mass\ of\ reactants - mass\ of\ products) \times c^2$

$$= (M_{Th} - M_{Ra^*} - M_\alpha) \times c^2$$

$$= [228.028726 - 224.02042 - 4.00260] \times 931\ MeV$$

$$= 5.3038\ MeV$$

18. Question

Calculate the maximum kinetic energy of the beta particle emitted in the following decay scheme:

$^{12}N \rightarrow {}^{12}C^* + e^+ + v$

$^{12}C^* \rightarrow {}^{12}C + \gamma$ (4.43 MeV).

The atomic mass of ^{12}N is 12.018613 u.

Answer

Given: $^{12}N \rightarrow {}^{12}C^* + e^+ + v$

$^{12}C^* \rightarrow {}^{12}C + \gamma$

Adding the two reactions:

$$N^{12} \rightarrow C^{12} + e^+ + v + \gamma(4.43MeV)$$

Max K.E of β-particle ={[mass of ^{12}N – mass of ^{12}C] × c^2} –4.43Mev

$$= [12.018613 - 12] \times 931 - 4.43$$

$$= 12.89\ MeV$$

Note: for max K.E β-particle, K.E of ν is considered zero

19. Question

: The decay constant of ${}^{197}_{80}Hg$ (electron capture to ${}^{197}_{79}Au$) is $1.8 \times 10^{-4}\ s^{-1}$.

(a) What is the half-life?

(b) What is the average-life?

(c) How much time will it take to convert 25% of this isotope of mercury into gold?

Answer

a) Half-life $= \frac{\ln 2}{decay\ constant}$

$$= \frac{ln\,2}{1.8 \times 10^{-4}}\ s$$

$$= 3850.81\ s$$

$$\approx 64\ min.$$

b) Average-life $= \frac{1}{decay\ constant}$

$$= \frac{1}{1.8 \times 10^{-4}}\ s$$

$$= 5555.56\ s$$

$$\approx 92\ min.$$

c) We know,

$$\frac{A}{A_0} = \left(\frac{1}{2}\right)^n$$

where A= Activity of the substance

A_0 = Initial activity

N = number of half lives

According to the problem,

Present activity = (1-0.25) A_0

= $0.75A_0$

So,

$$\frac{0.75A_0}{A_0} = \left(\frac{1}{2}\right)^n$$

$$\ln 0.75 = n \times ln\left(\frac{1}{2}\right)$$

$$N = 0.415 \times 3850.81s$$

$$= 1598.23\ s$$

20. Question

The half-life of ^{198}Au is 2.7 days.

(a) Find the activity of a sample containing 1.00 μg of ^{198}Au.

(b) What will be the activity after 7 days? Take the atomic weight of ^{198}Au to 198 g mol^{-1}.

Answer

a) 198 g of Au contains 6.023×10^{23} atoms (Avogadro's Number)

So 1μg of Au contains $\frac{6.023\times10^{23}}{198} \times 10^{-6}$ atoms

$= 3.041 \times 10^{15}$ atoms

Activity = λN , λ = decay constant = $\frac{\ln 2}{half\ life}$

$$= \frac{\ln 2 \times 3.041 \times 10^{15}}{2.7 \times 24 \times 3600} \frac{distintegrations}{second}$$

$$= \frac{\ln 2 \times 3.041 \times 10^{15}}{2.7 \times 24 \times 3600 \times 3.7 \times 10^{10}} Ci$$

$$= 0.244\ Ci$$

b) We know,

$$\frac{A}{A_0} = \left(\frac{1}{2}\right)^n$$

where A= Activity of the substance

A_0 = Initial activity

N = number of half lives

Here $n = \frac{7}{2.7}$

$= 2.592$

$A = 0.5^{2.592} \times Ao$

$= 0.5^{2.592} \times 0.244\ Ci$

$= 0.0404\ Ci$

21. Question

Radioactive ^{131}I has a half-life of 8.0 days. A sample containing ^{131}I has activity 20 μ Ci at t = 0.

(a) What is its activity at t = 4.0 days?

(b) What is its decay constant at t = 4.0 days?

Answer

Given,

Half-life = 8 days, A_0 = 20μCi

a) We know that,

$$A = A_0 e^{-\lambda t}$$

$$\lambda = \frac{\ln 2}{half\ life}$$

Where,

A= Activity of the substance

A_0 = Initial activity

t = time

λ = decay constant

So,

$$\lambda = \frac{ln\,2}{8}$$

$$\lambda = 0.086\ day^{-1}$$

$$A = A_0 e^{-\lambda t}$$

$$A = 20 \times 10^{-6} \times e^{-0.086 \times 4}$$

$$= 14.14\ \mu Ci$$

b) Decay constant in per second = $\frac{0.086}{24 \times 3600}$ s^{-1}

$= 1.4 \times 10^{-6} s^{-1}$

22. Question

The decay constant of ^{238}U is 4.9×10^{-18} s^{-1}.

(a) What is the average-life of ^{238}U?

(b) What is the half-life of ^{238}U?

(c) By what factor does the activity of a ^{238}U sample decrease in 9×10^9 years?

Answer

Given,

The decay constant of ^{238}U is 4.9×10^{-18} s^{-1}

a) Average life = $\frac{1}{decay\ constant}$

$= 2.04 \times 10^{17}\ s$

$= 6.49 \times 10^9\ years$

b) Half life = $\frac{\ln 2}{decay\ constant}$

$= 1.414 \times 10^{17}\ s$

$= 4.48 \times 10^9\ years$

c) 9×10^9 years = 2 half-lives(approximately)

We know

$$\frac{A}{A_0} = \left(\frac{1}{2}\right)^n$$

where A= Activity of the substance

A_0 = Initial activity

N = number of half lives

N = 2, in this case.

So it is evident that the sample activity will decrease by a factor of 4

23. Question

A certain sample of a radioactive material decays at the rate of 500 per second at a certain time. The count rate falls to 200 per second after 50 minutes.

(a) What is the decay constant of the sample?

(b) What is its half-life?

Answer

Initial Activity = 500

Final activity = 200

a) We know that,

$$A = A_0 e^{-\lambda t}$$

$$\lambda = \frac{\ln 2}{half\ life}$$

Where,

A= Activity of the substance

A_0 = Initial activity

t = time=50 mins

λ = decay constant

so,

$$200 = 500 \times e^{-50\times 60\times \lambda}$$

$$e^{-50\times 60\times \lambda} = \frac{2}{5}$$

$$-3000\lambda = \ln\left(\frac{2}{5}\right)$$

$$-3000\lambda = -0.9161$$

$$\lambda = 3.05 \times 10^{-4}\, s^{-1}$$

b) Half life $= \frac{\ln 2}{3.05\times 10^{-4}}$

$$= 2272.61\, s$$

$$= 38\, min$$

24. Question

The count rate from a radioactive sample falls from 4.0×10^6 per second to 1.0×10^6 per second in 20 hours. What will be the count rate 100 hours after the beginning?

Answer

We know that,

$$A = A_0 e^{-\lambda t}$$

$$\lambda = \frac{\ln 2}{half\ life}$$

Where,

A= Activity of the substance=$1 \times 10^6\ s^{-1}$

A_0 = Initial activity=$4 \times 10^6\ s^{-1}$

t = time=20h

λ = decay constant

So according to the problem,

$$1 \times 10^6 = 4 \times 10^6 \times e^{-\lambda \times 20h}$$

$$e^{-20\lambda} = 0.25$$

Now we have to evaluate activity after 100 hours

$$A = 4 \times 10^6 \times\ e^{-100\lambda}$$

$$A = 4 \times 10^6 \times \left(e^{-20\lambda}\right)^5$$

$$A = 4 \times 10^6 \times (0.25)^5$$

A = 3.9×10^3 disintegrations per second

Hence, the activity after 100 hours is 3.9×10^3 disintegrations per second.

25. Question

The half-life of ^{226}Ra is 1602 y. Calculate the activity of 0.1g of $RaCl_2$ in which all the radium is in the form of ^{226}Ra. Taken atomic weight of Ra to be 226 g mol^{-1} and that of Cl to be 35.5 g mol^{-1}.

Answer

Molecular weight of $RaCl_2 = 1(\ 226) + 2(35.5)\ = 297$

So, 297 g of $RaCl_2$ contains 6.023×10^{23} atoms

0.1 g contains $= \frac{6.023\times10^{23}\times0.1}{297}$

$= 2.027 \times 1020\ atoms(A)$

Now for decay constant(λ) $= \frac{\ln 2}{half\ life} = \frac{ln2}{1602\ years} = \frac{ln2}{5.052\times10^{10}}$

$= 1.37 \times 10^{-11}\ s^{-1}$

Activity = λA

where

A= no. of atoms

λ = decay constant

$activity = 1.37 \times 10^{-11} \times 2.027 \times 10^{20}$

$= 2.8 \times 10^{9} \frac{disintegrations}{s}$

26. Question

The half-life of a radioisotope is 10 h. Find the total number of disintegrations in the tenth hour measured from a time when the activity was 1 Ci.

Answer

We know that,

$A = A_0 e^{-\lambda t}$

$\lambda = \frac{\ln 2}{half\ life}$

Where,

A= Activity of the substance

A_0 = Initial activity

t = time

λ = decay constant

Given,

Half-life = 10h, A_0=1 Ci

Activity after 9 hours $= e^{\left(-\frac{ln2}{10}\right)\times 9}$

= 0.536 Ci

Number of atoms left after 9 hours $= \frac{Activity\ in\ 9\ hrs}{decay\ constant}$

$$= \frac{0.536 \times 3.7 \times 10^{10} \times 3600}{\frac{\ln 2}{10}}$$

$$= 1.03 \times 10^{15}\ atoms$$

Activity after 10 hours = 0.5 Ci [As 10h is half-life]

Number of atoms left after 10 hours $= \frac{Activity\ in\ 10\ hrs}{decay\ constant}$

$$= \frac{0.5 \times 10 \times 3.7 \times 10^{10} \times 3600}{\ln 2}$$

$$= 9.60 \times 10^{14}\ atoms$$

Hence, net disintegrations at the 10^{th} hour = $(1.03\times10^{15} - 9.60\times10^{14})$

= 6.92×10^{13} atoms

27. Question

The selling rate of a radioactive isotope is decided by its activity. What will be the second-hand rate of a one month old ^{32}P ($t_{1/2}$ = 14.3 days) source if it was originally purchased for 800 rupees?

Answer

Given, $t_{1/2}$ = 14.3 days

⇒The disintegration rate, $\lambda = \frac{ln2}{14.3} s^{-1}$

t = 1 month = 30 days

A_o = 800 disintegrations/sec

According to Law of Radioactivity, rate of disintegration of a radioactive isotope at time t will decay exponentially with rate of disintegration initially.

So, $A = A_o e^{-\lambda t}$

$$= 800e^{-\frac{0.693\times30}{14.3}}$$

$= 187$disintegrations/sec

Hence, selling rate will be 187 rupees.

28. Question

^{57}Co decays to ^{57}Fe by β^+-emission. The resulting ^{57}Fe is in its excited state and comes to the ground state by emitting γ-emission is 10^{-8} s. A sample of ^{57}Co gives 5.0×10^9 gamma rays per second. How much time will elapse before the emission rate of gamma rays drops to 2.5×10^9 per second?

Answer

Given, ^{57}Co decays by β^+ and γ- emission. Rate of emission of gamma rays is 5.0×10^9.

As the emission rate reduce to half of the given value. So the amount of ^{57}Co by β^+-emission should reduce to half of the original amount.

∵ Time elapsed for drop of emission rate to half is half life of the β^+-emission.

29. Question

Carbon (Z = 6) with mass number 11 decays to boron (Z = 5).

(a) Is it is β^+-decay or a β^--decay?

(b) The half-life of the decay scheme is 20.3 minutes. How much time will elapse before a mixture of 90% carbon-11 and 10% boron-11 (by the number of atoms) converts itself into a mixture of 10% carbon-11 and 90% boron-11?

Answer

Given, ^{6}C decays to ^{5}B.

(a) $^{11}{}_6C \rightarrow {}^{11}{}_5B + \beta^+$

So, it is β^+ decay.

(b) $t_{1/2}$ = 20.3 min.

$\lambda = \frac{ln2}{20.3}$ min^{-1}

$C_i = 0.9C_o$, $C_f = 0.1C_o$

$$C_f = C_i\, e^{-\frac{0.693 \times t}{20.3}}$$

So, $0.1C_o = 0.9C_o e^{-\frac{0.693 \times t}{20.3}}$

$$\Rightarrow e^{-\frac{0.693 \times t}{20.3}} = \frac{0.1}{0.9}$$

Taking $\log_e$ both sides, we get

$\Rightarrow -\frac{0.693 \times t}{20.3} = \ln\left(\frac{0.1}{0.9}\right)$

$\therefore -\frac{0.693 \times t}{20.3} = \ln\left(\frac{1}{9}\right)$

So, $t = \frac{20.3 \times 2.197}{0.693}$

$t = 64.36\, min.$

30. Question

4 × 10^{23} tritium atoms are contained in a vessel. The half-life of decay of tritium nuclei is 12.3 y. Find

(a) the activity of the sample,

(b) the number of decays in the next 10 hours

(c) the number of decays in the next 6.15 y.

Answer

Given, $t_{1/2}$ = 12.3 y

(a) Activity $= \frac{dN}{dt} = \lambda N = \frac{ln2}{12.3} \times 4 \times 1023\ \frac{dis}{y}$

$= 2.25 \times\ 1022\ dis/y$

(b) $\frac{dN}{dt} = 2.25 \times 10^{22} \frac{dis}{y}$

$= 2.57 \times 10\text{^}18\ \frac{dis}{h}$

$\therefore N = 2.57 \times 10^{18} \times 10$

= 2.57$\times$ 10^{19} atoms will decay in next 10 hours.

(c) No. of atoms initially, $N_{\circ} = 4 \times 10^{23}$

$N = N_{\circ}e^{\frac{-0.693 \times 6.15}{12.3}}$

$= 2.82 \times 1023$ atoms remained

So, No. of atoms decayed = $(4 - 2.82) \times 10^{23}$

$= 1.18 \times 10^{23}$

31. Question

A point source emitting alpha particles is placed at a distance of 1m from a counter which records any alpha particle falling on its 1 cm^2 window. If the source contains 6.0×10^{16} active nuclei and the counter records a rate of 50000 counts/second, find the decay constant. Assume that the source emits alpha particles uniformly in all directions and the alpha particles fall nearly normally on the window.

Answer

Given, Count received at 1m from source = 50000 counts/cm^2sec

As it is a point source, total nuclei radiated from the source

= Counts received per unit area × total area

$$= 4\pi r^2 \times 50000 \frac{counts}{sec}$$

So, $\frac{dN}{dt} = 4 \times 3.14 \times 1^2 \times 10^4 \times 50000$

$= 6.28 \times 10^9 \; counts/sec$

No. of active nuclei = $6 \times 10^{16} \; counts$

Now, $\frac{dN}{dt} = N\lambda$

$6.28 \times 10^9 = 6 \times 1016 \times \lambda$

$\therefore \lambda = 1.0467 \times 10^{-7} s^{-1}$

32. Question

^{238}U decays to ^{206}Pb with a half-life of 4.47×10^9 y. This happens in a number of steps. Can you justify a single half-life for this chain of processes? A sample of rock is found to contain 2.00 mg of ^{238}U and 0.600 mg of ^{206}Pb. Assuming that all the lead has come from uranium, find the life of the rock.

Answer

Half-life of any decay means the time taken to reduce the no. of atoms to half of the initial value. So, even for chain of the processes, half-life will be a unique value for a particular decay.

No. of atoms of ^{238}U= $\frac{6 \times 10^{23} \times 2 \times 10^{-3}}{238} = 0.0504 \times 10^{20}$

No. of atoms of ^{206}Pb = $\frac{6 \times 10^{23} \times 0.6 \times 10^{-3}}{206} = 0.017 \times 10^{20}$

Total no. of ^{238}U atoms initially = $(0.0504 + 0.017) \times 10^{20}$

$= 0.67 \times 10^{20}$

$$\lambda = \frac{ln2}{4.47 \times 10^9} = 0.1555 \times 10^{-9}$$

Now, $N = N_{\circ}e^{-\lambda t}$

$\Rightarrow 0.05 \times 10^{20} = 0.067 \times 10^{20} e^{-0.155 \times 10^{-9} t}$

Taking $\log_e$ on both sides, we get

$$ln\left(\frac{0.05}{0.067}\right) = -0.1555 \times 10^{-9} t$$

$\therefore t = 1.92 \times 10^9 y$

33. Question

When charcoal is prepared from a living tree, it shows a disintegration rate of 15.3 disintegrations of ^{14}C per gram per minute. A sample from an ancient piece of charcoal shows ^{14}C activity to be 12.3 disintegrations per gram per minute. How old is this sample? Half-life of ^{14}C is 5730 y.

Answer

Given, activity $A = 12.3 \frac{dis}{sec}$

Initial activity $A_{\circ} = 15.3 \frac{dis}{sec}$

Half-life of ^{14}C $t_{\frac{1}{2}} = 5730\ y$

Disintegration rate $\lambda = \frac{ln2}{5730}\ y^{-1}$

According to Law of Radioactivity,

No. of radioactive sample at time t,$N = N_{\circ}e^{-\lambda t}$

$\therefore 12.3 = 15.3 e^{-\frac{ln2}{5730}t}$

So, $\frac{12.3}{15.3} = e^{\frac{-ln2\ t}{5730}}$

Taking ln on both sides, we get

$\Rightarrow \ln\left(\frac{12.3}{15.3}\right) = \frac{-ln2\ t}{5730}$

So, $t = \frac{5730 \times 0.218}{ln2}$

$\Rightarrow t = 1804.2\ y$

34. Question

Natural water contains a small amount of tritium (^3_1H). This isotope beta-decays with a half-life of 12.5 years. A mountaineer while climbing towards a difficult peak finds debris of some earlier unsuccessful attempt. Among other things he finds a sealed bottle of whisky. On return 1.5 per cent of the ^3_1H radioactivity as compared to a recently purchased bottle marked '8 years old'. Estimate the time of that unsuccessful attempt.

Answer

Let initial activity of the bottle = A_o

Activity of the bottle found on the mountain, $A = A_o e^{-\lambda t}$

Also, $A = A_o e^{-\frac{ln2}{12.5}\times 8} \times 0.015$

Comparing both we get,

$$A_o e^{-\frac{ln2}{12.5}\times 8} \times 0.015 = A_o e^{-\lambda t}$$

$$-\frac{ln2 \times 8}{12.5} + ln(0.015) = -\frac{ln2}{12.5}t$$

$$(t-8)\frac{ln2}{12.5} + ln(0.015) = 0$$

$$\Rightarrow t = 83.74\, y$$

35. Question

The count rate of nuclear radiation coming from a radioactive sample containing ^{128}I varies with time as follows.

Time t (minute)	0	25	50	75	100
Count rate $R(10^9 s^{-1})$:	30	16	8.0	3.8	2.0

(a) Plot $ln(R_0/R)$ against t.

(b) From the slope of the best straight line through the points, find the decay constant λ.

(c) Calculate the half-life $t_{1/2}$.

Answer

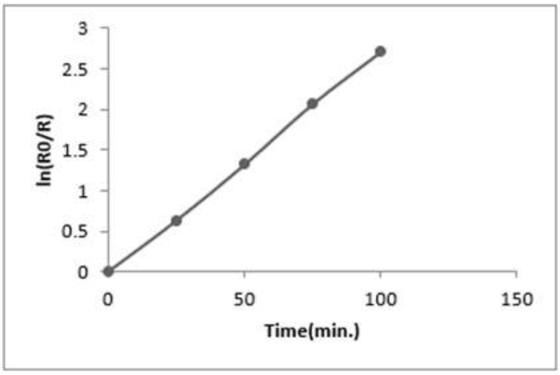
We take R_o at time $t=t_o$ i.e. $R_o = 30\times 10^9\ s^{-1}$

(a) $ln(\frac{R_0}{R_1}) = ln(\frac{30\times10^9}{30\times10^9}) = 0$

$$ln(\frac{R_0}{R_2}) = ln(\frac{30\times10^9}{16\times10^9}) = 0.63$$

$$ln(\frac{R_0}{R_3}) = ln(\frac{30\times10^9}{8\times10^9}) = 1.32$$

$$ln(\frac{R_0}{R_4}) = ln(\frac{30\times10^9}{3.8\times10^9}) = 2.067$$



$$ln(\frac{R_0}{R_5}) = ln(\frac{30\times10^9}{2\times10^9}) = 2.71$$

(b) As $\ln(R_0/R) = \lambda t$

So, slope of this curve will give the value of λ

$\therefore \lambda = 0.027\ \text{min}^{-1}$

(c) $t_{\frac{1}{2}} = \frac{ln2}{\lambda} = 25.67min.$

36. Question

The half-life of ^{40}K is 1.30×10^9 y. A sample of 1.00 g of pure KCl gives 160 counts s^{-1}. Calculate the relative abundance of ^{40}K (fraction of ^{40}K present) in natural potassium.

Answer

Given, $t_{1/2} = 1.3\times 10^9$ y

Activity, A= 160 counts s^{-1}

$$= 160 \times 3600 \times 24 \times 365 \frac{counts}{y}$$

As $A = \lambda N$

$$N = \frac{160 \times 3600 \times 24 \times 365 \times 1.3 \times 10^9}{ln2} counts = 9.46 \times 10^{18}$$

As 6.023×10^{23} atoms are present in 40g

$$\Rightarrow 1atom = \frac{40}{6.023 \times 10^{23}}$$

$$9.46 \times 10^{18} atoms = \frac{40 \times 9.46 \times 10^{18}}{6.023 \times 10^{23}} = 0.00063$$

∴ The relative abundance of ^{40}K in natural potassium= (2×0.00063×100) % = 0.12%

37. Question

$^{197}_{80}Hg$ decays to $^{197}_{79}Au$ through electron capture with a decay constant of 0.257 per day.

(a) What other particle or particles are emitted in the decay?

(b) Assume that the electron is captured from the K shell. Use Mosley's law $\sqrt{v} = \alpha (Z - b)$ with $\alpha = 4.95 \times 10^7\ s^{-1/2}$ and b = 1 to find the wavelength of the K_α X-ray emitted following the electron capture.

Answer

$^{197}{}_{80}Hg \rightarrow {}^{197}{}_{79}Au$

(a) A proton is converted to a neutron; a neutrino is emitted.

(b) As given: By Mosley's law, $\sqrt{v} = \alpha(Z - b)$

$$\sqrt{\frac{c}{\lambda}} = 4.95 \times 10^7 \times (79 - 1) = 4.95 \times 10^7 \times 78$$

$$\frac{3 \times 10^8}{\lambda} = (4.95 \times 10^7 \times 78)2$$

$$\therefore \lambda = 20pm$$

38. Question

A radioactive isotope is being produced at a constant rate dN/dt = R in an experiment. The isotope has a half-life $t_{1/2}$. Show that after a time t >> $t_{1/2}$, the number of active nuclei will become constant. Find the value of this constant.

Answer

Given, rate of radioactive isotope production= R

Rate of decay= λN (∵ According to law of Radioacivity)

As activity decreases exponentially and after a time t >> $t_{1/2}$, the number of active nuclei will become constant and rate of decay will also become constant.

So, $(dN/dt)_{produce} = (dN/dt)_{decay} = R$

R= λN

$\Rightarrow N = \frac{R}{\lambda} = \frac{Rt1/2}{ln2}$

39. Question

Consider the situation of the previous problem. Suppose the production of the radioactive isotope starts at t = 0. Find the number of active nuclei at time t.

Answer

Let $N_{\circ}$ is the radioactive isotope present at time t=0

N be the radioactive isotope present at time t

And λ be the disintegration constant.

By Law of Radioactivity,

$N = N_{\circ} e^{-\lambda t}$

No. of particles decay = $N_{\circ} - N = N_{\circ}(1 - e^{-\lambda t})$

As the production starts at t=0.

So, $\left(\frac{dN}{dt}\right)_{produce} = \left(\frac{dN}{dt}\right)_{decay} = R$

Activity $A_{\circ} = \lambda N_{\circ} = R$

$\Rightarrow N_{\circ} = \frac{R}{\lambda}$

$$N = N_{\circ}(1 - e^{-\lambda t}) = \frac{R}{\lambda}(1 - e^{-\lambda t})$$

40. Question

In an agricultural experiment, a solution containing 1 mole of a radioactive material ($t_{1/2}$ = 14.3 days) was injected into the roots of a plant. The plant was allowed 70 hours to settle down and then activity was measured in its fruit. If the activity measured was 1 μCi, what per cent of activity is transmitted from the root to the fruit in steady state?

Answer

n= 1mole = 6×10^{23} atoms=$N_{\circ}$

Given, $\lambda = \frac{ln2}{t^1/_2} = \frac{ln2}{14.3} per\ day = \frac{ln2}{(14.3\times24)} per\ hour$

According to Law of Radioactivity,

No. of radioactive sample at time t, $N = N_{\circ}e^{-\lambda t} = 6.023 \times 10^{23} e^{\frac{-ln2\times70}{14.3\times24}}$

N= 5.2×10^{23} atoms

According to law of Radioactivity, activity of a radioactive isotope at time t is the no. of active nuclei at that time times disintegration constant of the decay process.

As $\frac{dN}{dt} = \lambda N$

$= 2.9 \times 10^{17} \frac{dis}{s}$

Activity transmitted $= \frac{1\times3.7\times10^{4}}{2.9\times10^{17}} \times 100\%$

$= 1.27 \times 10^{-11}\%$

41. Question

A vessel of volume 125 cm^3 contains tritium (^3H, $t_{1/2}$ = 12.3 y) at 500 kPa and 300 K. Calculate the activity of the gas.

Answer

Given: Pressure P= 500000 Pa= 5 atm

Volume V= 0.125L

Temperature T= 300K

Assuming the gas to be ideal, according to ideal gas equation,

$PV = nRT$ (R be universal gas constant equal to $0.082 atmLmol^{-1}K^{-1}$, P is the pressure, V is the volume, T is the temperature)

So, $n = \frac{PV}{RT} = \frac{5\times0.125}{0.082\times300} = 0.0254$

$N = n \times 6 \times 10^{23} = 1.5 \times 10^{22}\ atoms$

$\lambda = \lambda = \frac{ln2}{t\,^1/_2} = \frac{ln2}{12.3} y^{-1}$

Activity, $A = \lambda N$

$= \frac{ln2}{12.3} \times 1.5 \times 10^{22} \frac{dis}{y}$

$= 8.4 \times 10^{20} \frac{dis}{y}$

42. Question

$^{212}_{83}Bi$ can disintegrate either by emitting an α-particle or by emitting a β^--particle.

(a) Write the two equations showing the products of the decays.

(b) The probabilities of disintegration by α-and β-decays are in the ratio 7/13. The overall half-life of ^{212}Bi is one hour. If 1g of pure ^{212}Bi is taken at 12.00 noon, what will be the composition of this sample at 1 p.m. the same day?

Answer

(a) $^{212}{}_{83}Bi \rightarrow {}^{208}{}_{81}Ti + \alpha$

$^{212}{}_{83}Bi \rightarrow {}^{212}{}_{84}Bi + \beta^-$

(b) $t_{1/2}$= 1h

After 1h, $^{212}{}_{83}Bi$ will be half decayed

So, $^{212}{}_{83}Bi$ is 0.5g present

$^{208}{}_{81}Ti$ and $^{212}{}_{84}Bi$ will be formed in the ratio 7/13 and total mass of the sample should be 1g. So, total mass of $^{208}{}_{81}Ti$ and $^{212}{}_{84}Bi$ is 0.5g.

⇒ Mass of $^{208}{}_{81}Ti$ will be ratio of mass present × total mass= $\frac{7\times0.5}{7+13} = 0.175g$

And mass of $^{212}{}_{84}Bi$ will be of mass present × total mass= $\frac{13\times0.5}{7+13} = 0.325g$

43. Question

A sample contains a mixture of ^{110}Ag and ^{108}Ag isotopes each having an activity of 8.0×10^8 disintegrations per second. ^{108}Ag is known to have larger half-life than

^{110}Ag. The activity A is measured as a function of time and the following data are obtained.

Time (s)	Activity (A) (10^8 disintegrations s^{-1})	Time (s)	Activity (A) (10^8 disintegrations s^{-1})
20	11.799	200	3.0828
40	9.1680	300	1.8899
60	7.4492	400	1.1671
80	6.2684	500	0.7212
100	5.4115		

(a) Plot $\ln(A/A_0)$ versus time.

(b) See that for large values of time, the plot is nearly linear. Deduce the half-life of ^{108}Ag from this portion of the plot.

(c) Use the half-life of ^{108}Ag to calculate the activity corresponding to ^{110}Ag in the first 50 s.

(d) Plot $\ln(A/A_0)$ versus time for ^{110}Ag for the first 50 s.

(e) Find the half-life of ^{108}Ag.

Answer

At time t=0, $A_\circ = 8.0 \times 10^8$ dis/s

(a) $\ln\left(\frac{A1}{A0}\right) = \ln\left(\frac{11.799}{8}\right) = 0.389) = 0.389$

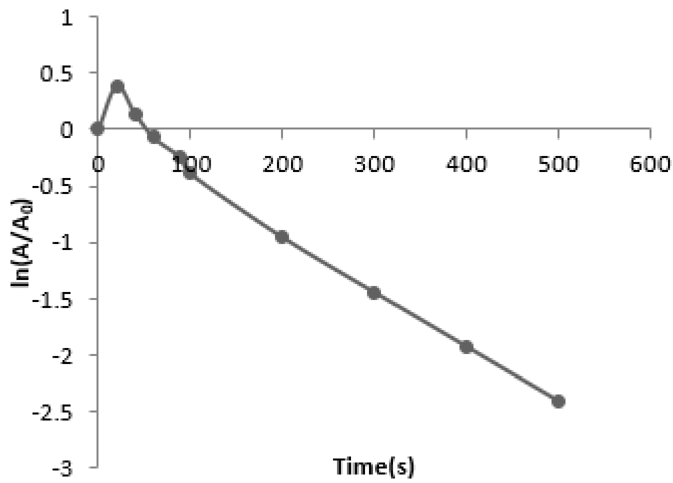
$$\ln\left(\frac{A2}{A0}\right) = \ln\left(\frac{9.168}{8}\right) = 0.136$$

$$\ln\left(\frac{A3}{A0}\right) = \ln\left(\frac{7.4492}{8}\right) = -0.0713$$

$$\ln\left(\frac{A4}{A0}\right) = \ln\left(\frac{6.2684}{8}\right) = -0.244$$

$$\ln\left(\frac{A5}{A0}\right) = \ln\left(\frac{5.4115}{8}\right) = -0.391$$

$$\ln\left(\frac{A6}{A0}\right) = \ln\left(\frac{3.0828}{8}\right) = -0.954$$



$$\ln\left(\frac{A7}{A0}\right) = \ln\left(\frac{1.8899}{8}\right) = -1.443$$

$$\ln\left(\frac{A8}{A0}\right) = \ln\left(\frac{1.1671}{8}\right) = -1.925$$

$$\ln\left(\frac{A9}{A0}\right) = \ln\left(\frac{0.7212}{8}\right) = -2.406$$

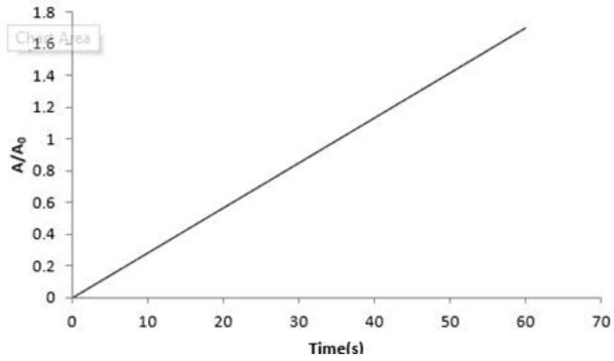
(b) For large values of time, the value of λ will be the slope negative of the slope of the curve.

∴ $\lambda = 0.028 s^{-1}$

So, $t_{1/2} = 24.4$ s

(c) $A = A_o e^{-\lambda t}$

(d) $8.0 \times 10^8 e^{-0.028\times 50} = 1.93 \times 10^8 dis/s$



(e) The half-life of ^{108}Ag from the graph is 144s.

44. Question

A human body excretes (removes by waste discharge, sweating, etc.) certain materials by a law similar to radioactivity. If technetium is injected in some form in a human body, the body excretes half the amount in 24 hours. A patient is given an injection containing ^{99}Tc. This isotope is radioactive with a half-life of 6 hours. The activity from the body just after the injection is 6 μCi. How much time will elapse before the activity falls to 3 μCi?

Answer

Given, t_1=24h and t_2=6h.

As both reactions are occurring parallel,

So, $\frac{1}{(t^1/_2)net} = \frac{1}{t1} + \frac{1}{t2}$

$$(t\,^1/_2)_{net}\,\frac{24 \times 6}{24 + 6} = 4.8h$$

$A_\circ$= 6 μCi and A=3 μCi

$\therefore ln2 = \lambda t$

$$ln2 = \frac{ln2}{4.8}\,t$$

$\Rightarrow t = 4.8h$

45. Question

A charged capacitor of capacitance C is discharged through a resistance R. A radioactive sample decays with an average-life τ. Find the value of R for which the ratio of the electrostatic field energy stored in the capacitor to the activity of the radioactive sample remains constant in time.

Answer

Let charge at time t be Q and initial charge be q. Q is given by Q= $q_{e\frac{-t}{CR}}$

And according to Law of Radioactivity,

$A = A_\circ e^{-\lambda t}$

So, $\frac{Energy}{Activity} = \frac{q^2 e^{\frac{-2t}{CR}}}{2C \times A_\circ e^{-\lambda t}}$

(where C is the capacitance, R is the resistance, t is the time, q is the charge, λ is the decay constant)

As the ratio should be independent of time,

$\therefore \frac{2}{CR} = \lambda$

$R = \frac{2\tau}{C}$

46. Question

Radioactive isotopes are produced in a nuclear physics experiment at a constant rate dN/dt = R. An inductor of inductance 100 mH, a resistor of resistance 100 Ω and a battery are connected to form a series circuit. The circuit is switched on at the instant the production of radioactive isotope starts. It is found that i/N remains constant in time where i is the current in the circuit at time t and N is the number of active nuclei at time t. Find the half-life of the isotope.

Answer

Given, Resistance R= 100Ω

Inductance L= 100mH

Let initial current be $i_\circ$

After time t,

Current at time t is given by $i = i_\circ(1 - e^{-\frac{tR}{L}})$

$N = N_\circ e^{-\lambda t}$

(where R is the resistance, L is the inductance)

So, $\frac{i}{N} = \frac{i_\circ(1-e^{-\frac{tR}{L}})}{N_\circ e^{-\lambda t}}$

As the ratio should be independent of time,

$\therefore \frac{R}{L} = \lambda$

$$10^3 = \frac{ln2}{t_{\frac{1}{2}}}$$

$t_{1/2} = 6.93 \times 10^{-4}\ s$

47. Question

Calculate the energy released by 1g of natural uranium assuming 200 MeV is released in each fission event and that the fissionable isotope ^{235}U has an abundance of 0.7% by weight in natural uranium.

Answer

Given, 1g of sample contains 0.007g ^{235}U

And 235g of ^{235}U contains 6.023× 10^{23} atoms

0.007g contains $\frac{6.023\times10^{23}}{235}$× 0.007 atoms

1 atom releases 200MeV energy

So, total energy= Energy released per atom× total no. of atoms

$$= \frac{6.023 \times 10^{23} \times 0.007 \times 200 \times 10^{6} \times 1.6 \times 10^{-19}}{235} J = 5.74 \times 10^{-8} J$$

48. Question

A uranium reactor develops thermal energy at a rate of 300 KW. Calculate the amount of ^{235}U being consumed every second. Average energy released per fission is 200 MeV.

Answer

Let n atoms consume per sec.

Total energy released per sec= No. of atoms consumed per sec × Energy released per atom

$= (n \times 200 \times 10^{6} \times 1.6 \times 10^{-19})$J

$300 \times 10^{3} = 3.2 \times n \times 10^{-11}$

n= 9.375× 10^{15}

235g of ^{235}U contains 6.023× 10^{23} atoms

9.375× 10^{15} atoms are present in $\frac{235\times9.375\times 10^{15}}{6.023\times10^{23}}$= 3.65× 10^{-6}g

49. Question

A town has a population of 1 million. The average electric power needed per person is 300W. A reactor is to be designed to supply power to this town. The efficiency with which thermal power is converted into electric power is aimed at 25%.

(a) Assuming 200 MeV of thermal energy to come from each fission event on an average, find the number of events that should take place every day.

(b) Assuming the fission to take place largely through ^{235}U, at what rate will the amount of ^{235}U decrease? Express your answer in kg per day.

(c) Assuming that uranium enriched to 3% in ^{235}U will be used, how much uranium is needed per month (30 days)?

Answer

(a) Energy radiated per fission= $2 \times 10^8 \times 1.6 \times 10^{(-19)} V$

$= 3.2 \times 10^{(-11)} \frac{J}{s}$

Usable energy or efficient energy per fission= $3.2 \times 10^{-11} \times \frac{25}{100} = 8 \times 10^{-12} \frac{J}{s}$

Total energy needed= $3 \times 10^8 \frac{J}{s}$

No. of fission per sec $= \frac{total\ energy}{efficient\ energy\ per\ fission} = \frac{3 \times 10^8}{8 \times 10^{-12}} = 0.375 \times 10^{20}$

No. of fissions per day= $0.375 \times 10^{20} \times 3600 \times 24 = 324 \times 10^{22}$

(b) No. of atoms disintegrated per day= 324×10^{22}

235g of ^{235}U contains 6.023×10^{23} atoms

324×10^{22} contains $\frac{235 \times 324 \times 10^{22}}{6.023 \times 10^{23}} = 1.264 \frac{kg}{day}$

(c) ^{235}U needed= 1.264kg/day that is 3% of uranium sample.

So, uranium needed per day= $\frac{126.4}{3} \frac{kg}{day}$

Uranium needed per month= $1264\ kg$

50. Question

Calculate the Q-values of the following fusion reactions:

(a) $^2_1H + ^2_1H \rightarrow ^3_1H + ^1_1H$

(b) $^2_1H + ^2_1H \rightarrow ^3_2He + n$

(c) $^2_1H + ^3_1H \rightarrow ^4_2He + n$

Atomic masses are $m(^2_1H)$ = 2.014102 u, $m(^3_1H)$ = 3.016049 u, $m(^3_2He)$ = 3.016029 u, $m(^4_2He)$ = 4.002603 u.

Answer

(a) $^2_1H + ^2_1H \rightarrow ^3_1H + ^1_1H$

Q value= $[2m(^2_1H) - (m(^3_1H) + m(^1_1H)]u$

$= [2 \times 2.014102 - (3.016049 + 1.007835)] \times 931MeV$

$= 4.05MeV$

(b) $^2_1H + ^2_1H \rightarrow ^3_2He + n$

Q value= $[2m(^2_1H) - (m(^3_2He) + m(n]u$

$= [2 \times 2.014102 - (3.016029 + 1.008665)] \times 931MeV$

$= 3.25MeV$

(c) $^2_1H + ^3_1H \rightarrow ^4_2He + n$

Q value= $[m(^2_1H) + m(^3_1H) - (m(^4_2He) + m(n)]u$

$= [\,2.014102 + 3.016049 - (4.002603 + 1.008665)] \times 931MeV$

$= 17.57MeV$

51. Question

Consider the fusion in helium plasma. Find the temperature at which the average thermal energy 1.5 kT equals the Coulomb potential energy at 2 fm.

Answer

According to electrostatic potential energy,

$PE = \frac{kz \times z}{r}$ (where k is 9×10^9 N m^2 C^{-2}, r is the distance, Z is the charge)

$$= \frac{9 \times 10^9 \times (2 \times 1.6 \times 10^{-19}) \times (2 \times 1.6 \times 10^{-19})}{2 \times 10^{-15}}$$

$= 4.6 \times 10^{-13} J$

$1.5KT = 1.5 \times 1.38 \times 10 - 23 \times T = 4.6 \times 10^{-13}$

$\Rightarrow T = 2.23 \times 10^{10} K$

52. Question

Calculate the Q-value of the fusion reaction

$^4He + ^4He = ^8Be$.

Is such a fusion energetically favorable? Atomic mass of ^{8}Be is 8.0053 u and that of ^{4}He is 4.0026 u.

Answer

$^{4}He + {}^{4}He = {}^{8}Be$.

Q value= $[2m(^{4}He)- m(^{8}Be)]u$

= [2×4.0026 – 8.0053] × 931MeV

=-93.1KeV

Negative sign indicates that the energy has to be provided to proceed this reaction. So, this fusion is not favorable.

53. Question

Calculate the energy that can be obtained from 1 kg of water through the fusion reaction

$^{2}H + {}^{2}H \rightarrow {}^{3}H + p$.

Assume that $1.5 \times 10^{-2}\%$ of natural water is heavy water D_2O (by number of molecules) and all the deuterium is used for fusion.

Answer

Given, 18g of molecules contain 6.023×10^{23}atoms

1kg of molecules contain $\frac{6.023\times10^{23}\times1000}{18} = 3.346 \times 10^{25}$ atoms

% of deuterium atoms=No. of atoms in 1kg of water× % of deuterium

$= 3.346 \times 10^{25} \times 99.985$

Energy of deuterium$= 3.346 \times 10^{25} = (4.028204 - 3.016044) \times 931eV$

$= 942.32 \times 1.6 \times 10^{-19}\,J$

= 1507mJ