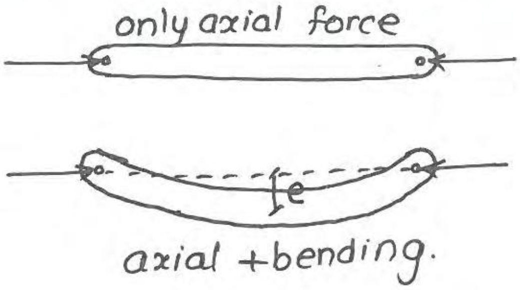
3. Statically Determinate Truss

3.1 Need of Truss:

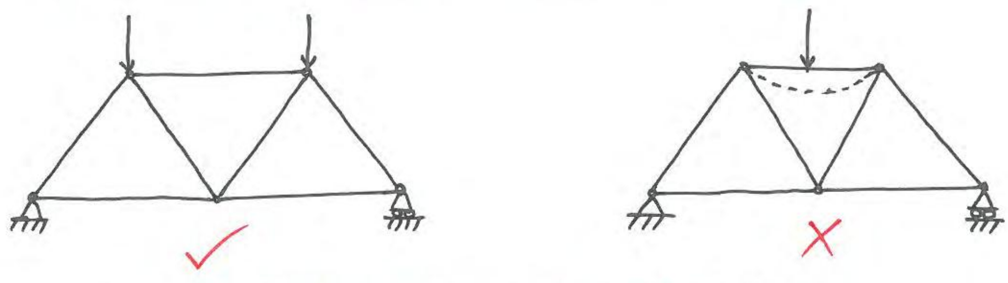
- As span increases, BM also increases so depth requirement for beam is very high. In this situation, truss becomes better option
- Since all members of truss are axially loaded so all fibres are equally stressed. In beams, extreme fibres are more stressed as compared to inner fibres so truss becomes economical than beam.

3.2 Assumptions:

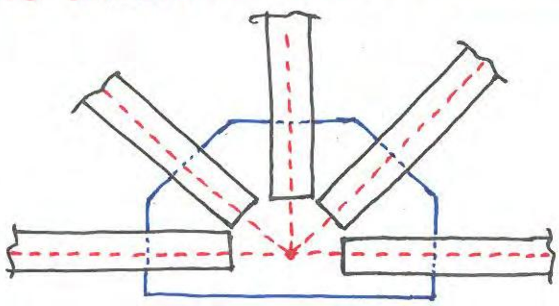
1) Members are perfectly straight.



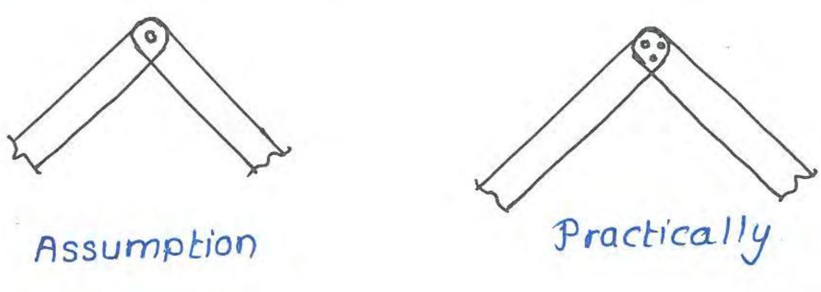
2) Loading must be applied at joints only.



3) Members should be concurrent at joints.

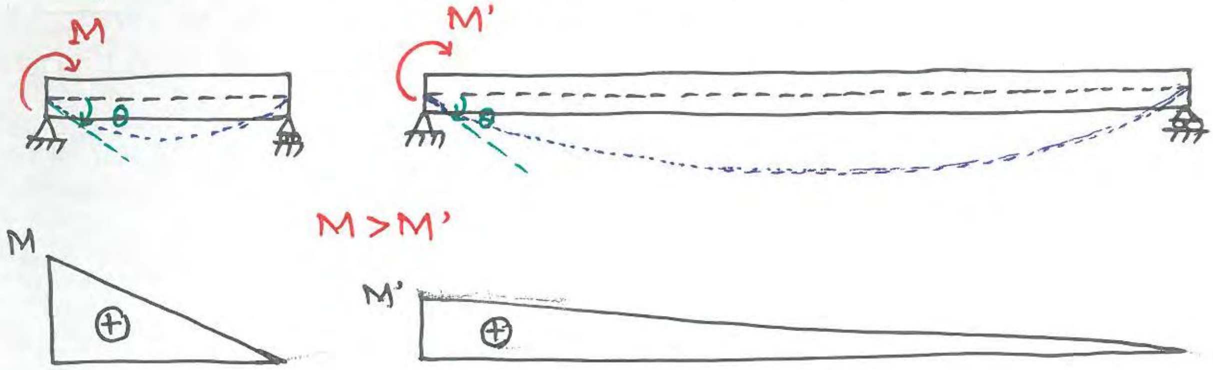


4) All joints are frictionless and pin connected.

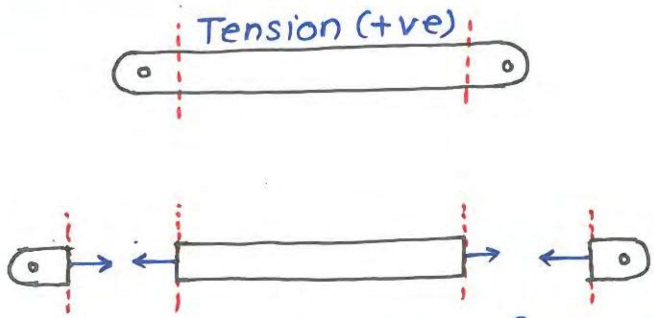


* Note:

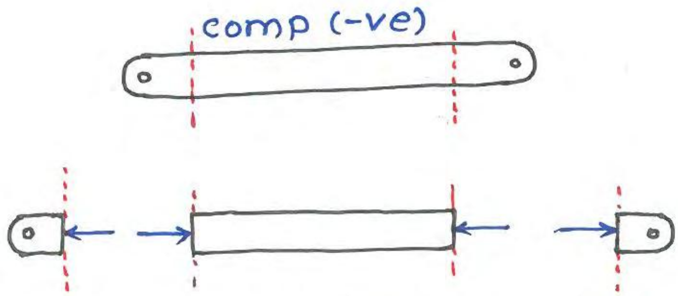
Practically all joints of truss are rigid so rotation of joint produces BM in members. Since members of truss are very slender so BM due to rotation of joints is very less. That is why this BM is neglected in analysis.



3.3 Sign Convention:



Force is always away from cross-section.

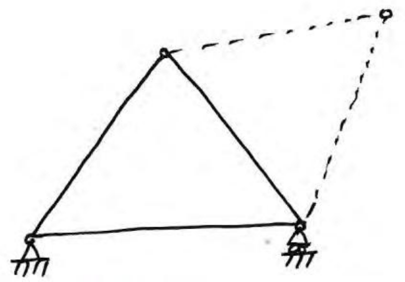


Force is always towards cross-section.

- Types of Truss:-

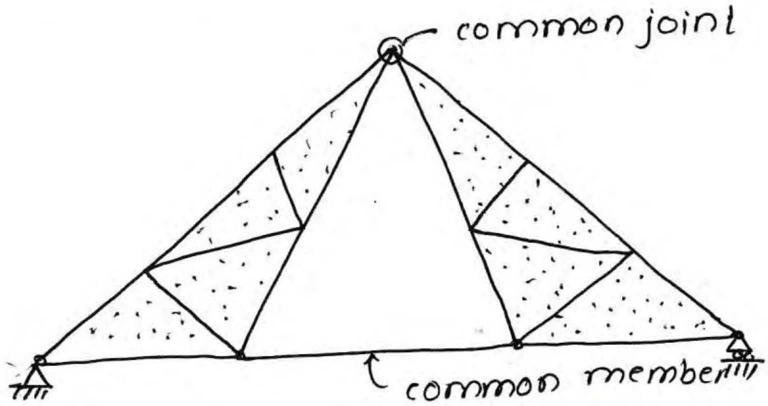
1) Simple Truss:-

If two members are required to make the additional joint in simple triangular truss then such truss is classified as simple truss.

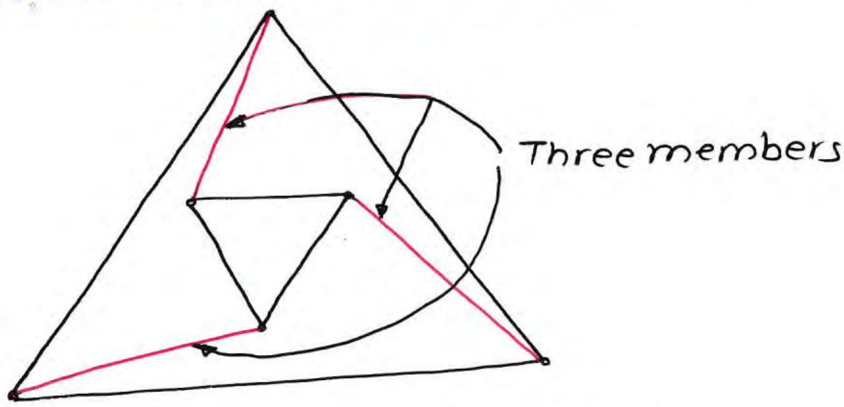


2) Compound Truss:-

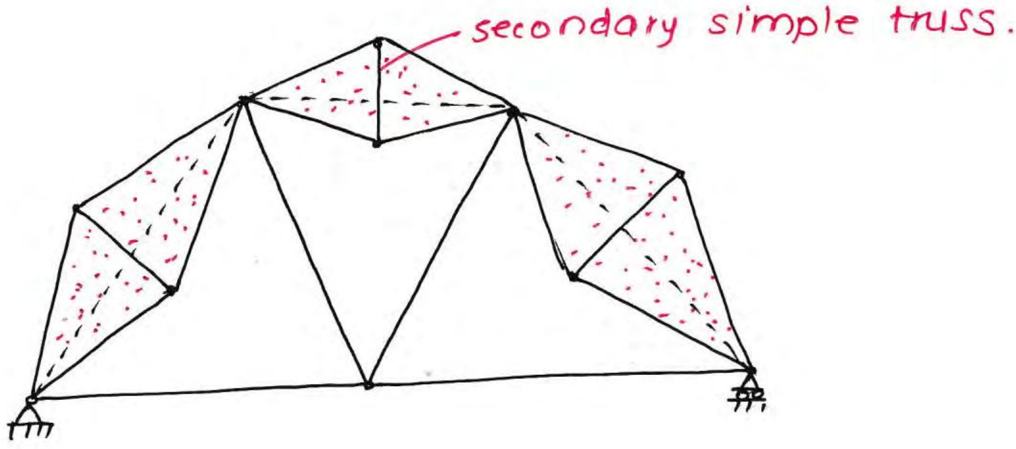
a) If 2 simple trusses are joined by a common joint and a common member.



b) If three members are used to join two simple trusses.

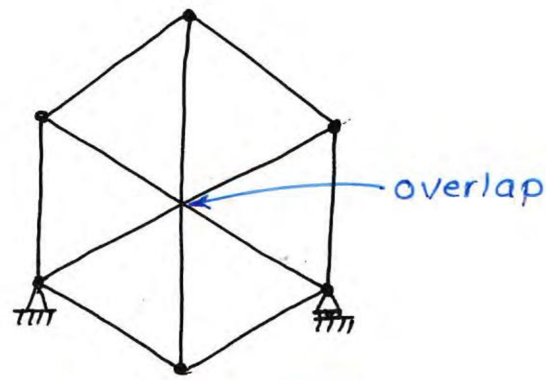


c) If any member of simple truss is replaced by secondary simple truss:-



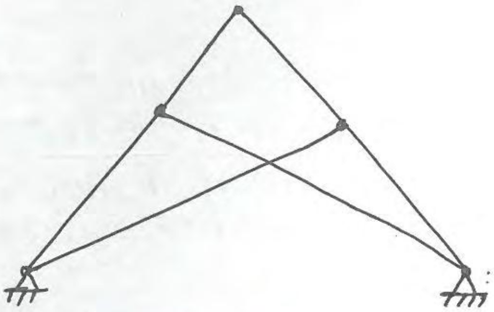
3) Complex truss:-

Trusses that can not be classified as simple and compound is called complex truss.

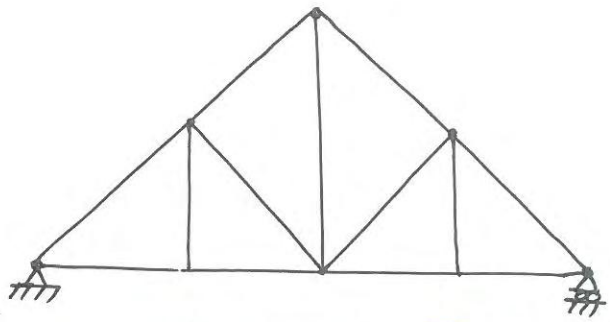


Complex Truss.

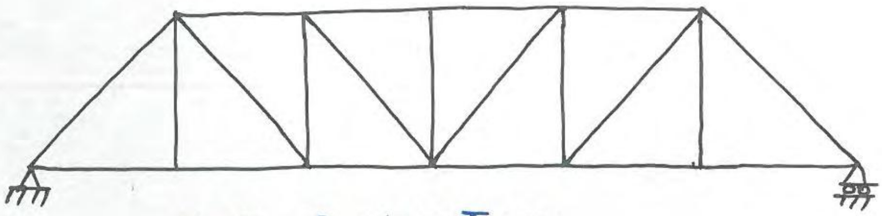
3.4 Types of Truss:



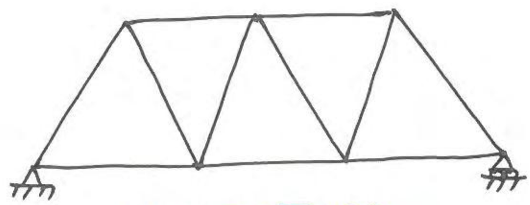
Scissor Truss



Pratt Building Truss



Pratt Bridge Truss



Warren Truss.

3.5 Methods of Analysis.

1) Method of Joint
2) Method of Section
3) Graphical Method
4) Tension Coefficient Method.

*Note:
Willot Mohr's method is a graphical method which is used for calculation of deflection of truss.

3.5.1 Method of Joints:

Step I: Calculate support reactions using overall equilibrium of truss.

Step II: Identify joint of sart in such a way that number of unknowns at that joint should not be more than 2.

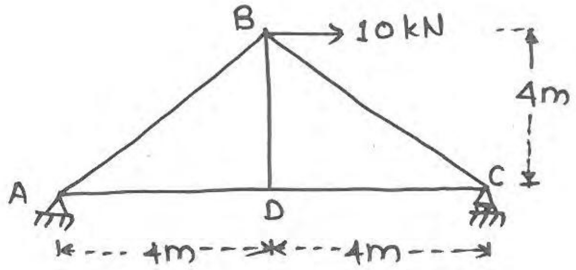
Step III: Apply equations of equilibrium at joint of step II ($\Sigma F_x=0$ and $\Sigma F_y=0$) and calculate unknown forces at that joint.

Step IV: Move to another joint where again not more than 2 unknowns are present.

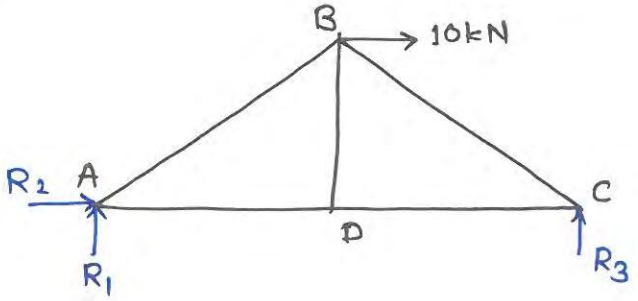
Step V: Repeat step III and step IV till force in all members are known.

Step VI: Represent member force in tabular formate.

Ex. 1.



⇒



$\Sigma F_x=0$

$\Rightarrow R_2+10=0$ ----(i)

$R_2=-10$

$\Sigma F_y=0$

$\Rightarrow R_1+R_3=0$ ------(ii)

$\Sigma M_z=0$

$\Rightarrow \Sigma M_A=0$

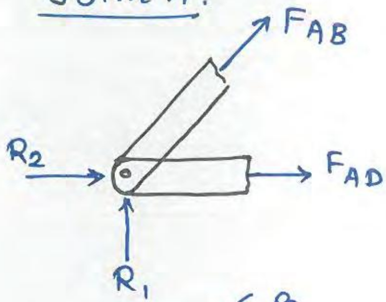
$\Rightarrow 10\times 4-R_3\times 8=0$

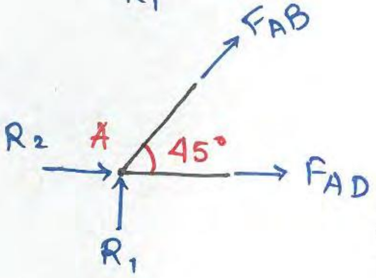
$R_3=5\,kN$ ·····(iii)

from equation (ii)

$R_1=-5\,kN$

Joint A:





$\Sigma F_x = 0$

$\Rightarrow R_2 + F_{AB} \cos 45 + F_{AD} = 0$

$\Rightarrow -10 + F_{AB} \times \frac{1}{\sqrt{2}} + F_{AD} = 0 \quad \cdots (iv)$

$\Sigma F_y = 0$

$\Rightarrow R_1 + F_{AB} \sin 45 = 0$

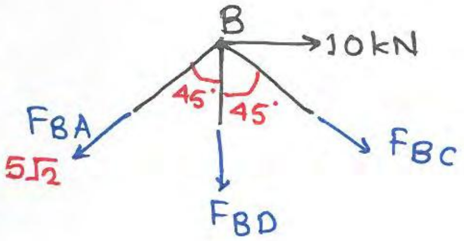
$\Rightarrow -5 + F_{AB} \times \frac{1}{\sqrt{2}} = 0 \quad \cdots\cdots (v)$

From eq^n (iv) & (v)

$F_{AB} = 5\sqrt{2}$ kN

$F_{AD} = 5$ kN.

Joint B:



$\Sigma F_x = 0$

$\Rightarrow -F_{BA} \sin 45° + F_{BC} \sin 45° + 10 = 0$

$\Rightarrow -5\sqrt{2} \times \frac{1}{\sqrt{2}} + F_{BC} \times \frac{1}{\sqrt{2}} + 10 = 0$

$\Rightarrow \quad F_{BC} = -5\sqrt{2}$

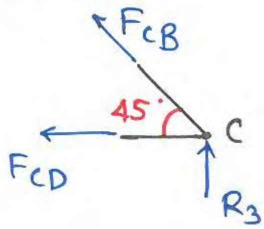
$\Sigma F_y = 0$

$\Rightarrow -F_{BA} \cos 45° - F_{BD} - F_{BC} \cos 45° = 0$

$\Rightarrow -(5\sqrt{2}) \times \frac{1}{\sqrt{2}} - F_{BD} - (-5\sqrt{2}) \times \frac{1}{\sqrt{2}} = 0$

$\Rightarrow \quad F_{BD} = 0$

Joint C:



$\Sigma F_x = 0$

$-F_{CD} - F_{CB} \sin 45 = 0$

$\Rightarrow -F_{CD} - (-5\sqrt{2}) \times \frac{1}{\sqrt{2}} = 0$

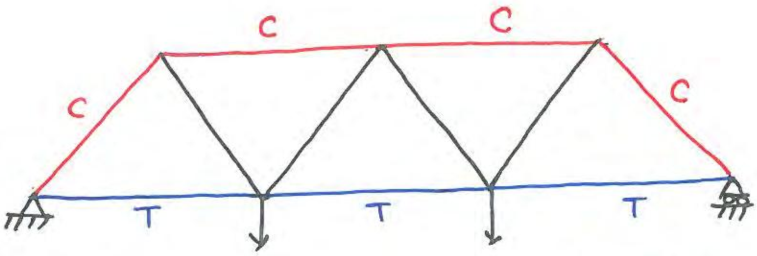
$\Rightarrow F_{CD} = 5$ kN.

Member		Force (kN)
AB	⟶	$5\sqrt{2}$
AD	⟶	5
BD	⟶	0
BC	⟶	$-5\sqrt{2}$
CD	⟶	5

(+) → Tension

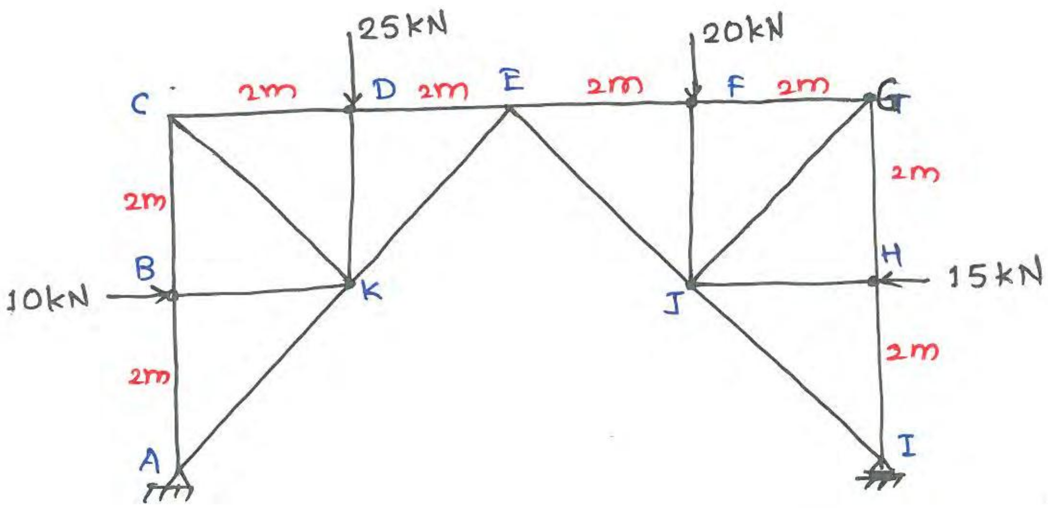
* Note:

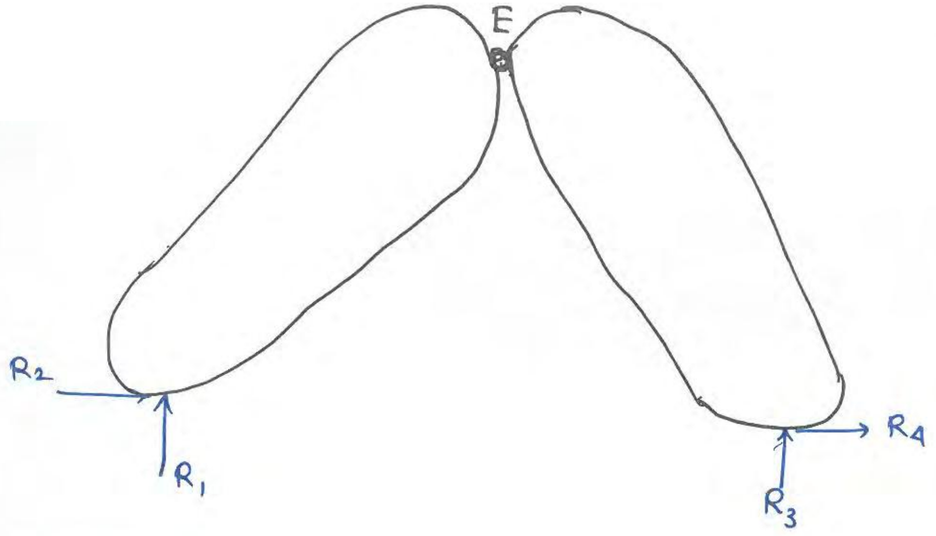
IF bridge truss is subjected to downward loading then tope chord and bottom chord members are subjected to compression and tension respectively.



Top chord → compression

Bottom chord → Tension.





Equations:

$\Sigma F_x = 0$

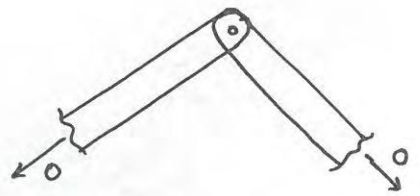
$\Sigma F_y = 0$

$\Sigma M_z = 0$

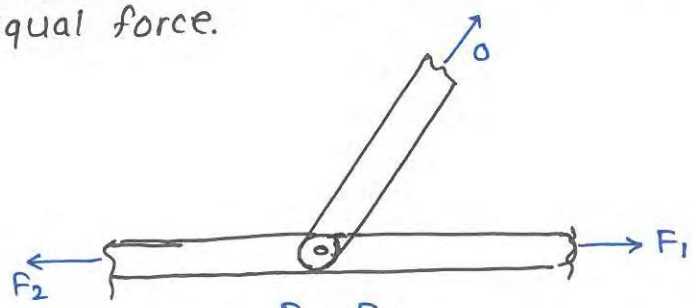
$M_E = 0$ (R.H.S.)

3.5.2 Some Tricks:

Trick 1: If two non collinear members are meeting at a joint and subjected to no external force or reaction then both members have zero force

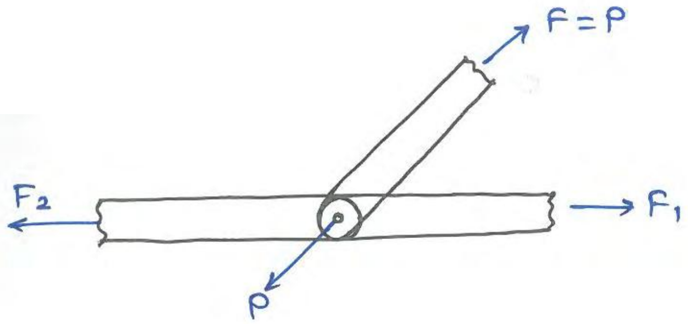


Trick 2: If three members are meeting at a joint out of which two are collinear and joint is subjected to no external force or reaction then non-collinear member has zero force and collinear members have equal force.

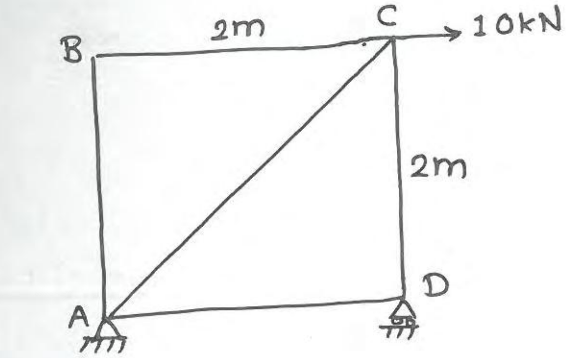


$F_1 = F_2$

Trick 3: If three members are meeting at a joint out of which two are collinear and 1 external force or reaction is along non-collinear member then non-collinear member has force equal to applied loading or reaction.

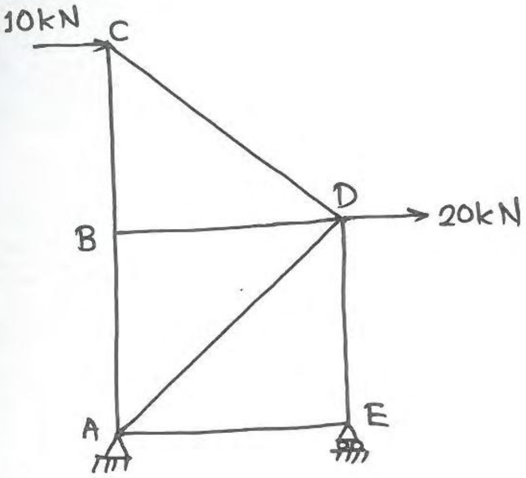


$F_1 = F_2$



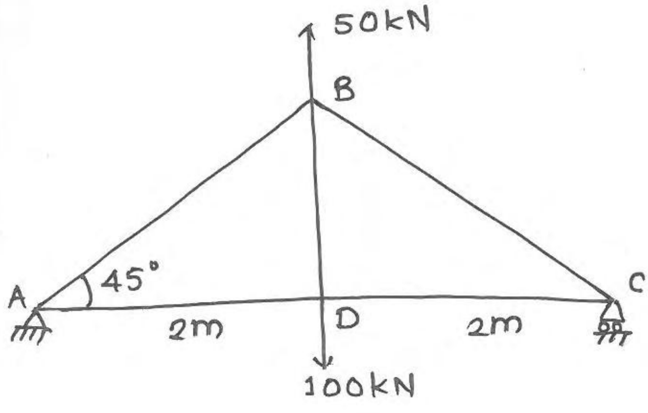
$F_{BC} = ?$

$F_{BC} = 0$



$F_{BD} = ?$

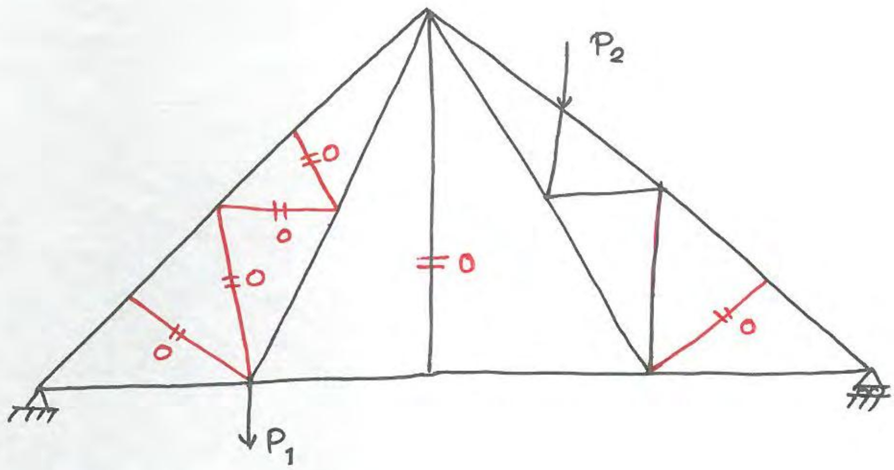
$F_{BD} = 0$



$F_{BD} = ?$

$F_{BD} = 100\ kN$

Q. How many members have zero force ?



Note:

Zero force member is provided to reduce effective length of other members.

3.5.3 Method of Section

Procedure:

Step I: Calculate support reactions if required.

Step II: Decide cut section

Step III: Use equations of equilibrium to calculate force in desired member.

Guidelines to decide cut Section:

i) Section should pass through the member in which force is to be calculated.

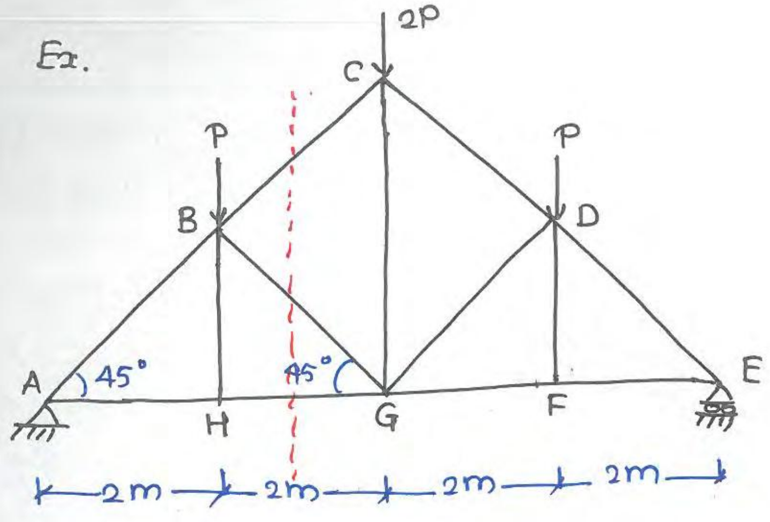
~~ii) Section should cut minimum number of members to calculate force in desired m~~

ii) Section may cut any number of members.

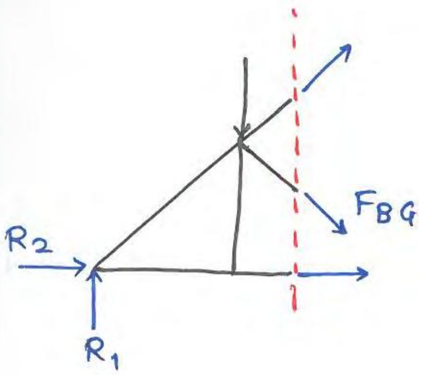
iii) Section should be such that force in desired member can be calculated using either of following equations.

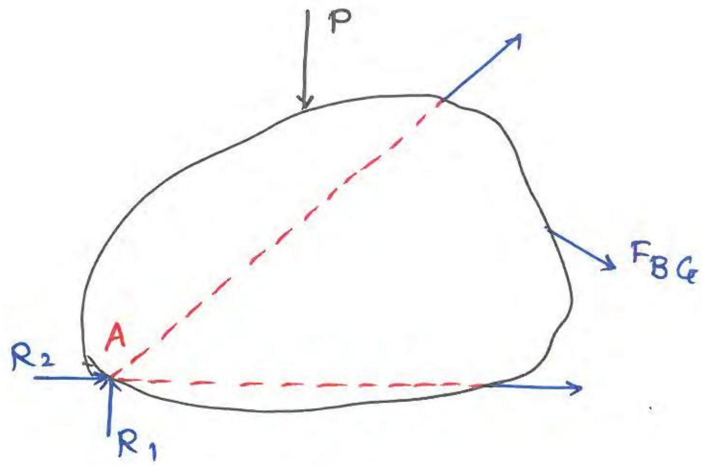
$$\Sigma F_x = 0,\ \Sigma F_y = 0,\ \text{and}\ \Sigma M_z = 0.$$

Ex.



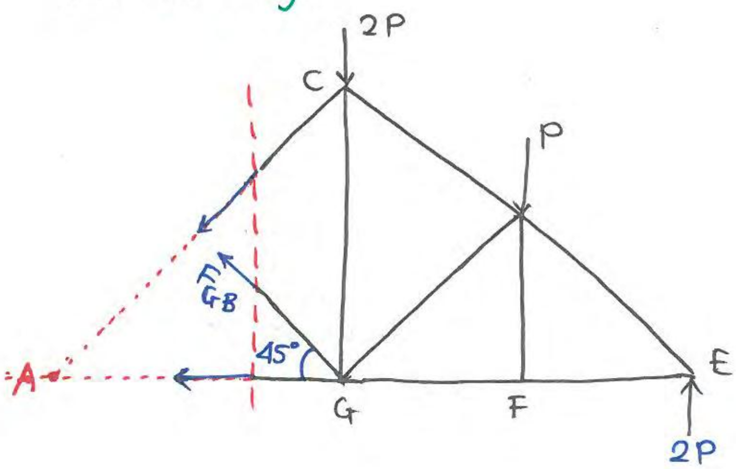
$F_{BG} = ?$





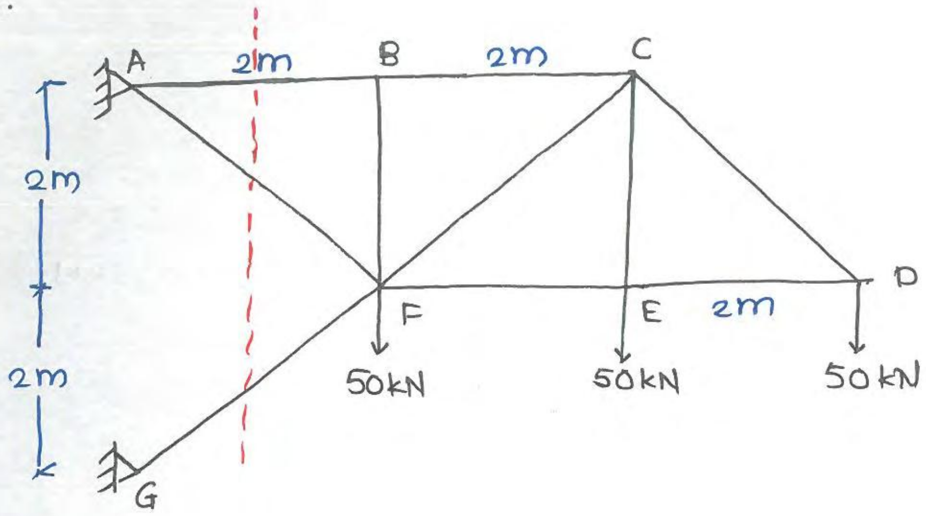
$$\Sigma M_z = 0$$
$$\Rightarrow \Sigma M_A = 0$$
$$\Rightarrow \quad P \times 2 + F_{BG} \times 2\sqrt{2} = 0$$
$$F_{BG} = \frac{-P}{\sqrt{2}}$$

Alternatively:

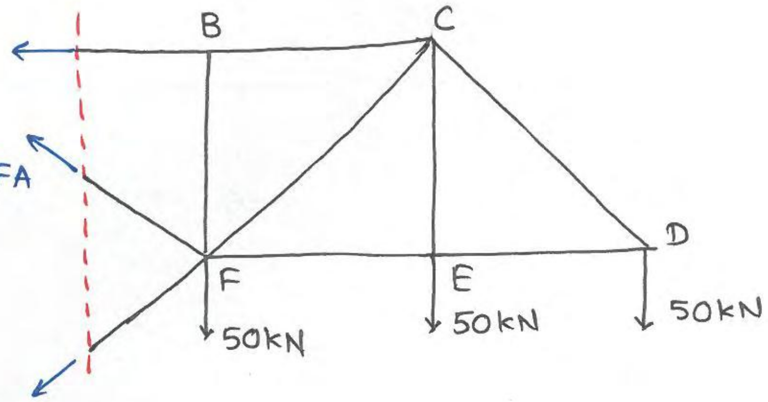


$$\Sigma M_z = 0$$
$$\Rightarrow \Sigma M_A = 0$$
$$\Rightarrow \quad 2P \times 4 + P \times 6 - 2P \times 8$$
$$-(F_{GB} \sin 45^\circ) \times 4 = 0$$
$$\Rightarrow F_{GB} = -P/\sqrt{2}$$

Ex. 2.



$F_{AF} = ?$



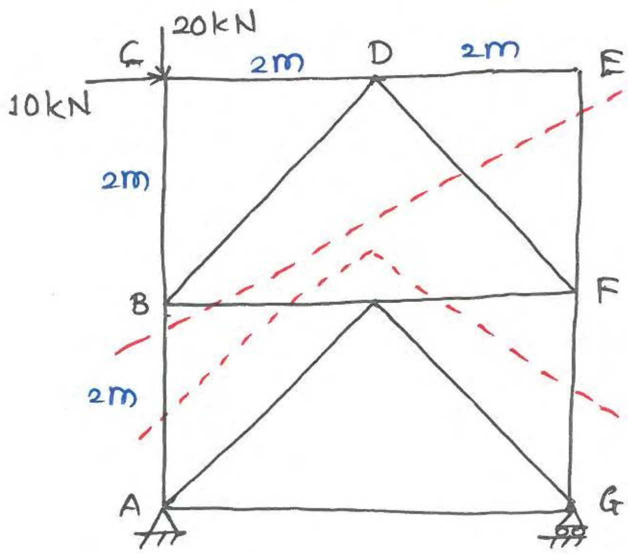
$\Sigma M_z = 0$

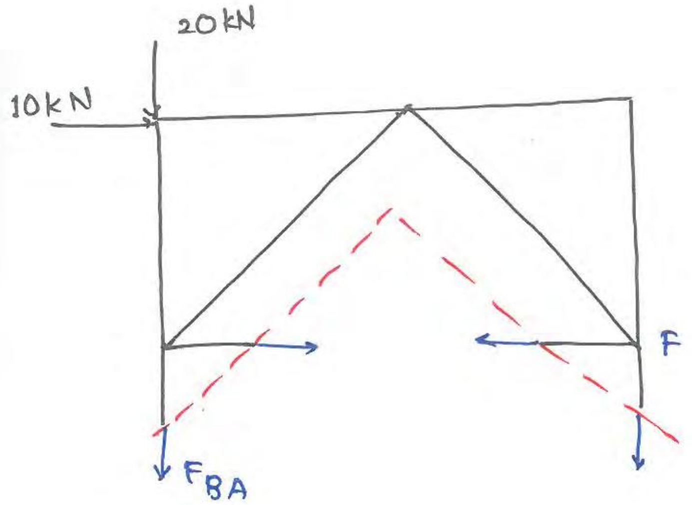
$\Rightarrow \Sigma M_C = 0$

$-50 \times 2 + 50 \times 2 + F_{FA} \times 2\sqrt{2} = 0$

$F_{FA} = 0$

Ex. 3.





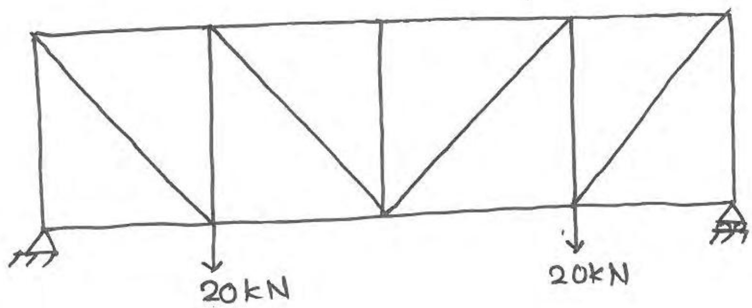
$\Sigma M_z = 0$

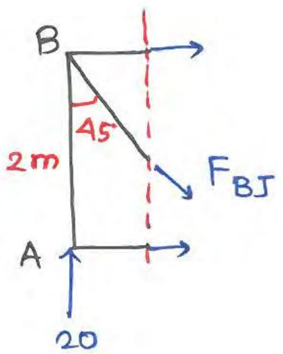
$\Rightarrow \Sigma M_F = 0$

$\Rightarrow -20\times4 + 10\times2 - F_{BA}\times4 = 0$

$F_{BA} = -15\,kN$

Ex.





$\Sigma F_y = 0$

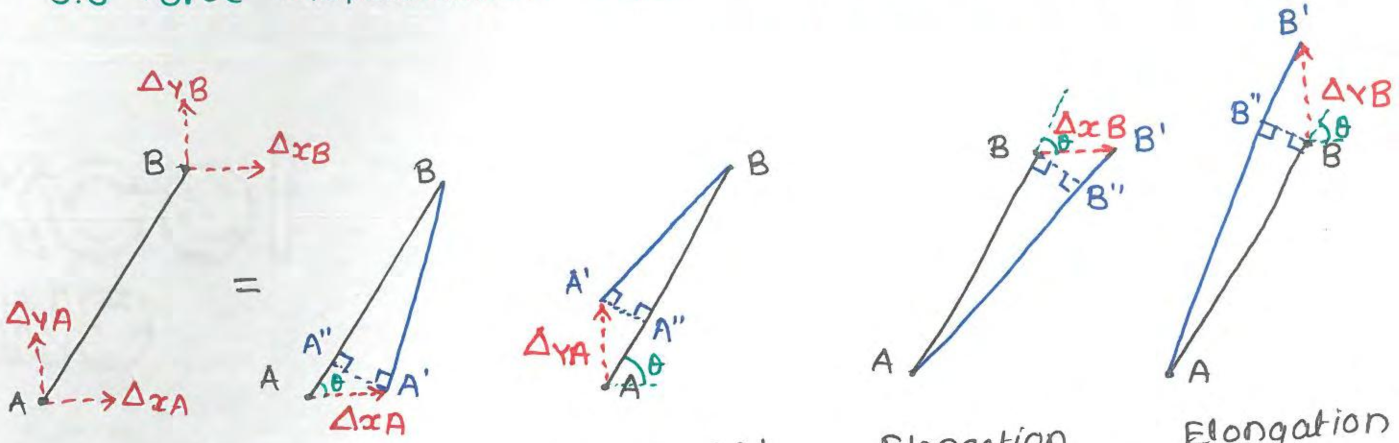
$20 - F_{BJ}\cos45° = 0$

$F_{BJ} = 20\sqrt{2}\ kN$

* Note:

- If all unknown forces of FBD are parallel except a force which needs to be calculated then either $\Sigma F_x = 0$ or $\Sigma F_y = 0$ is used.
- If all unknown forces of FBD are concurrent at any point except a force which need to be calculated then $\Sigma M_z = 0$ is used.

3.6 Force Displacement Equation of Axial Member:



Comp = AA'

$= \Delta_{xA} \cos\theta$

= component of Δ_{xA} along AB.

comp = AA'

$= \Delta_{yB} \sin\theta$

Elongation = BB'

$= \Delta_{xB} \cos\theta$

Elongation = BB'

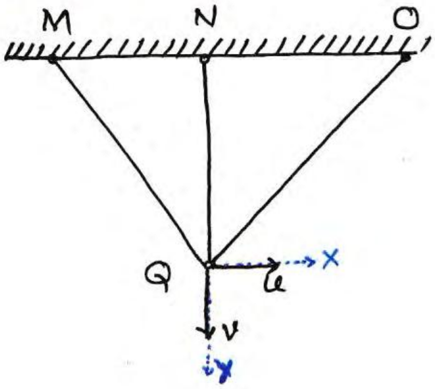
$= \Delta_{YB} \sin\theta$

Considering elongation as +ve

$$F_{AB} = \frac{AE}{L} \Delta$$

$$= \frac{AE}{L} \left(\Delta_{xB} \cos\theta + \Delta_{YB} \sin\theta - \Delta_{xA} \cos\theta - \Delta_{YA} \sin\theta \right)$$

Q4 In a redundant joint model, three bar members are pin connected at Q as shown in the figure. Under some load placed at Q, the elongation of members MQ and OQ are found to be 48 mm and 35 mm respectively. Then the horizontal displacement 'u' and the vertical displacement 'v' of the node Q, in mm, will be respectively



MN = 400 mm

NO = 500 mm

NQ = 500 mm

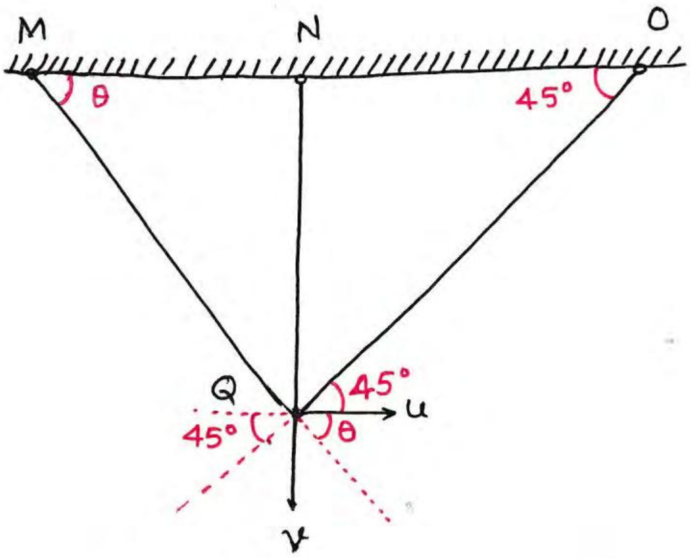
a) −6.64 and 56.14

b) 6.64 and 56.14

c) 0.0 and 59.41

d) 59.41 and 0.0

[2003 : 2 marks]



Taking components of u and v along members,

$$u\cos\theta + v\sin\theta = 48 \quad \cdots (i)$$

$$-u\cos 45^\circ + v\sin 45^\circ = 35 \quad \cdots (ii)$$

from (i) and (ii)

$u = 6.65$ mm

$v = 56.14$ mm.

$$\cos\theta = \frac{400}{\sqrt{400^2+500^2}}$$

$$\sin\theta = \frac{500}{\sqrt{400^2+500^2}}$$