Fourier Series, Energy and Signals

2

Fourier Series

Fourier Representation of Periodic Function

$$\boxed{f(t) = a_0 + 2\sum_{n=1}^{\infty}\left[a_n \cos n\omega_0 t + b_n \sin n\omega_0 t\right]}$$

where $f(t)$ = Periodic function

$\omega_0 = \dfrac{2\pi}{T_0}$ = Fundamental frequency

T_0 = Fundamental period of function f(t)

$$\boxed{a_0 = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} f(t)dt}$$ ......mean value of function

$$\boxed{a_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} f(t)\cdot \cos n\omega_0 t\, dt}$$

$$\boxed{b_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} f(t)\cdot \sin n\omega_0 t\, dt}$$

Complex Exponential Fourier Series

$$\boxed{f(t) = \sum_{n=-\infty}^{\infty} c_n \exp(-jn\omega_0 t)}$$

Where,

$$\boxed{c_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} f(t)\cdot \exp(-jn\omega_0 t)\, dt}$$ for $n = 0, \pm 1, \pm 2 \ldots$

Remember:

Fourier series is used to represent a periodic signal where as Fourier transform is used to represent a non-periodic signal.

- For distortionless transmission

$$\boxed{y(t) = k\,g(t - t_d)}$$

where
$y(t)$ = Output
$g(t)$ = Input
k = Gain
t_d = Time delay

- Transfer function required for distortionless transmission

$$\boxed{H(\omega) = ke^{-j\omega t_d}}$$

Paley-Wiener criterion

The necessary and sufficient condition for amplitude response $|H(\omega)|$ to be realizable

$$\boxed{\int_{-\infty}^{\infty} \frac{ln|H(\omega)|}{1+\omega^2} d\omega < \infty}$$

- Energy contained in a given signal f(t)

$$\boxed{E = \int_{-\infty}^{\infty} f^2(t)dt}$$

Parseval's theorems

$$\boxed{E = \frac{1}{2\pi}\int_{-\infty}^{\infty} |F(\omega)|^2 d\omega}$$

- Energy spectral density

$$\boxed{G_E(\omega) = |F(\omega)|^2}\text{, J/Hz}$$

it is the energy contained in the signal per unit bandwidth

- Auto correlation function (ACF)

$$\boxed{R(\tau) = \int_{-\infty}^{\infty} f(t) \cdot f(t - \tau)d\omega}$$

■■■■