Short Answer Type Questions – I

[2 marks]

State true or false for each of the following and justify your answer (Q.1 to 3)

Que 1. AB is a diameter of a circle and AC is its chord such that $\angle BAC = 30°$ If the tangent at C intersects AB extended at D, then BC = BD.

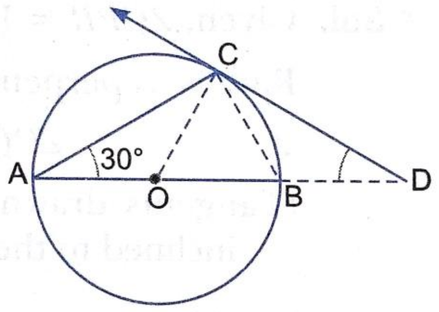

Fig. 8.16

Sol. True, Join OC,

$\angle ACB = 90°$ (Angle in semi-circle)

$\therefore \quad \angle OBC = 180° - (90° + 30°) = 60°$

Since, OB = OC = radii of same circle [Fig. 8.16]

$\therefore \quad \angle OBC = \angle OCB = 60°$

Also, $\angle OCD = 90°$

$\Rightarrow \quad \angle BCD = 90° - 60° = 30°$

Now, $\angle OBC = \angle BCD + \angle BDC$ (Exterior angle property)

$\Rightarrow \quad 60° = 30° + \angle BDC \quad \Rightarrow \quad \angle BDC = 30°$

$\because \quad \angle BCD = \angle BDC = 30° \quad \therefore \quad BC = BD$

Que 2. The length of tangent from an external point P on a circle with centre O is always less than OP.

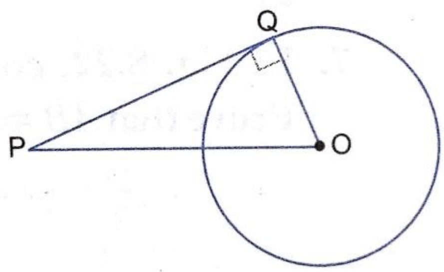

Fig. 8.17

Sol. True. Let PQ be the tangent from the external point P.

Then ΔPQO is always a right angled triangle with OP as the hypotenuse.

So, PQ is always less than OP.

Que 3. If angle between two tangents drawn from a point to a circle of radius 'a' and centre O is 90°, then OP = $a\sqrt{2}$.

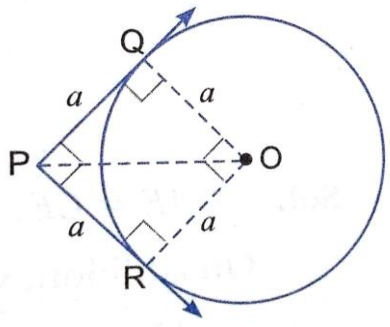

Fig. 8.18

Sol. True, let PQ and PR be the tangents
since $\angle P = 90°, so\ \angle QOR = 90°$
Also, OR = OQ = a
∴ $PQOR$ is a square

$$\Rightarrow\ OP = \sqrt{a^2 + a^2} = \sqrt{2a^2} = a\sqrt{2}$$

Que 4. In Fig. 8.19, PA and PB are tangents to the circle drawn from an external point P. CD is the third tangent touching the circle at Q. If PA = 15 cm, find the perimeter of ΔPCD.

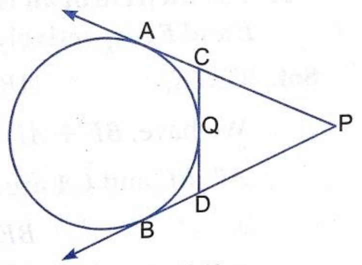

Fig. 8.19

Sol. ∵ PA and PB are tangent from same external point
∴ PA = PB = 15 cm
Now, perimeter of $\Delta PCD = PC + CD + DP = PC + CQ + QD + DP$
$= PC + CA + DB + DP$
$= PA + PB = 15 + 15 = 30\ cm$

Que 5. Prove that the line segment joining the points of contact of two parallel tangent of a circle, passes through its centre.

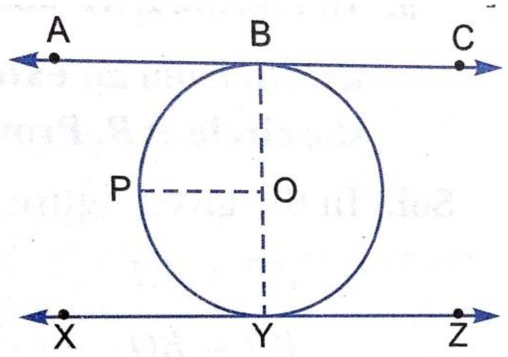

Fig. 8.20

Sol. Let the tangent to a circle with centre O be ABC and XYZ.

Construction: Join OB and OY.

Draw OP || AC

Since AB || PO

$\angle ABO + \angle POB = 180°$ (Adjacent interior angles)

$\angle ABO = 90°$ (A tangent to a circle is perpendicular to the radius through the point of contact)

$\Rightarrow$ $90° + \angle POB = 180°$ $\Rightarrow$ $\angle POB = 90°$

Similarly $\angle POY = 90°$

$\therefore$ $\angle POB + \angle POY = 90° + 90° = 180°$

Hence, BOY is a straight line passing through the centre of the circle.

Que 6. If from an external point P of a circle with centre O, two tangents PQ and PR are drawn such that ∠QPR = 120°, prove that 2 PQ = PO.

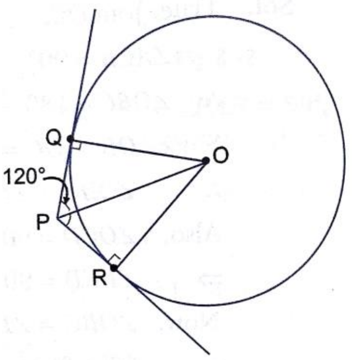

Fig. 8.21

Sol. Given, $\angle QPR = 120°$

Radius is perpendicular to the tangent at the point of contact.

$\therefore$ $\angle OQP = 90°$ $\Rightarrow$ $\angle QPO = 60°$

(Tangent drawn to a circle from an external point are equally inclined to the segment, joining the centre to that point)

In ΔQPO, $\cos 60° = \frac{PQ}{PO}$ $\Rightarrow \frac{1}{2} = \frac{PQ}{PO}$

$\Rightarrow$ $2\,PQ = PO$

Que 7. In Fig. 8.22, common tangent AB and CD to two circles with centres O_1 and O_2 intersect at E. Prove that AB = CD.

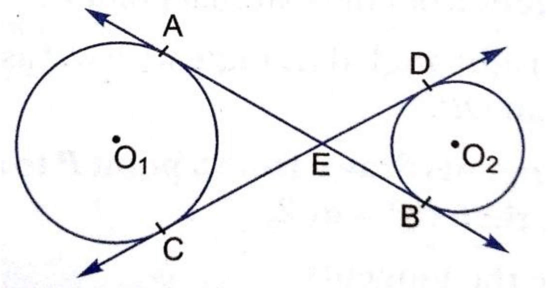

Fig. 8.22

Sol. AE = CE and BE = ED [Tangents drawn from an external point are equal]

On addition, we get
AE + BE = CE + ED $\Rightarrow$ AB = CD

Que 8. The incircle of an isosceles triangle ABC, in which AB = AC, touches the sides BC, CA and AB at D, E and F respectively. Prove that BD = DC.

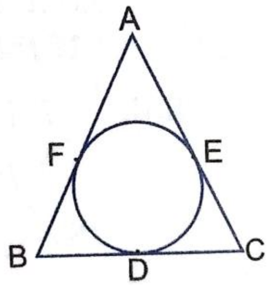

Fig. 8.23

Sol. Given, AB = AC
We have, BF + AF = AE + CE ….(i)
AB, BC and CA are tangent to the circle at F, D and E respectively.
$\therefore$ BF = BD and CE = CD ….(ii)

From (i) and (ii)
BD + AE = AE + CD ($\because$ AF = AE)
$\Rightarrow$ BD = CD