Polynomials

OLYMPIAD EXCELLENCE BOOK **MATHEMATICS**

QUESTIONS

1. **If** $x = (\sqrt{2}+1)^{-\frac{1}{3}}$ **the value of** $\left(x^3 \frac{1}{x^3}\right)$ **is**

(a) 0 (b) $-\sqrt{2}$ (c) $2\sqrt{2}$ (d) $3\sqrt{2}$

2. **If** $(3a+1)^2+(b-1)^2+(2c-3)^2=0$, than **value of** $(3a+b+2c)$ **is equal to:**

(a) 3 (b) -1 (c) 2 (d) 5

3. **If** $x^2+y^2+z^2+\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=6$, **then the valve of** $x^2+y^2+z^2$ **is**

(a) 3 (b) 4 (c) 8 (d) 16

4. **If** $x+y+z=0$ **then** $3\left[\frac{x^2}{yz}+\frac{y^2}{zx}+\frac{z^2}{xy}\right]=?$

(a) $(xyz)^2$ (b) $x^2+y^2+z^2$ (c) 9 (d) 3

5. **If** $x=3+2\sqrt{2}$ **and** $xy=1$, **then the value of** $\frac{x^2-3xy+y^2}{x^2+3xy+y^2}$ **is**

(a) $\frac{30}{31}$ (b) $\frac{70}{31}$ (c) $\frac{35}{31}$ (d) $\frac{31}{37}$

6. **If** $x=\sqrt{3}+\frac{1}{\sqrt{3}}$ **and** $y=\sqrt{3}-\frac{1}{\sqrt{3}}$, **then the value of** $\frac{x^2}{y}+\frac{y^2}{x}$ **is**

(a) $\sqrt{3}$ (b) $3\sqrt{3}$ (c) $16\sqrt{3}$ (d) $2\sqrt{3}$

7. **If** $a=\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$ **and** $b=\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$ **then the value of** $\frac{a^2-ab+b^2}{a^2+ab+b^2}=?$

(a) $\frac{63}{61}$ (b) $\frac{67}{65}$ (c) $\frac{65}{63}$ (d) $\frac{69}{67}$

8. **if** $a^4+b^4=x^2y^2$, **then** (a^6+b^6) **equals**

(a) 0 (b) 1 (c) x^2+y^2 (d) $a^2b^4+a^2b^2$

9. **If** $xy(x-y)=1$, **then the value of** $\frac{1}{x^3y^3}-x^3+y^3$ **is:**

(a) 0 (b) 1 (c) 3 (d) -3

10. **If** $\left(a+\frac{1}{a}\right)^2=3$, **then the value of** $a^{206}+a^{200}+a^{90}+a^{84}+a^{18}+a^{12}+a^6+1$ **is**

(a) 0 (b) 1 (c) 84 (d) 206

11. **If** $x+\frac{1}{x}=3$, **then the value of** $\frac{x^4+3x^3+5x^2+3x+1}{x^4+1}$

(a) 3 (b) 5 (c) 7 (d) 9

12. **If $x+y+z=6$, then the value of $(x-1)^3+(y-2)^3+(z-3)^3$ is**

(a) $3(x-1)(y+2)(z-3)$ (b) $3(x+1)(y-2)(z-3)$

(c) $3(x-1)(y-2)(z+3)$ (d) $3(x-1)(y-2)(z-3)$

13. **If $x=999$, then the value of $\sqrt[3]{x(x^2+3x+3)+1}$ is**

(a) 1000 (b) 999 (c) 998 (d) 1002

14. **The degree of the polynomial $\frac{x^4+x^5-x^8}{x^3}$ is**

(a) 2 (b) 3 (c) 4 (d) 5

15. **What is the remainder when $2x^2-3x+5$ is divided by $2x-1$?**

(a) 2 (b) 3 (c) 4 (d) 5

16. **2 is a root of $kx^4-13x^3+kx^2+12x$. What is the value of k?**

(a) $k=1$ (b) $k=2$ (c) $k=3$ (d) $k=4$

17. **If $x+1$ and $x-1$ are factors of $f(x)=x^4+3ax+b$, then the value of $3a+2b$ is**

(a) 5 (b) -1 (c) 4 (d) -6

18. **If the area of rectangle is $3x^2+6xy+3y^2$ and its breadth is $x+y$, then its length is**

(a) $x-2y$ (b) $-x+2y$ (c) $3x+3y$ (d) $x+y$

19. **If $(x+2)$ and $(x+3)$ are two factors of $x^3+9x^2+26x+24$, then the third factor is**

(a) $x+7$ (b) $x+9$ (c) $x+4$ (d) $x+8$

20. **If $x=2$, $y=3$ and $z=-5$, then $x^3+y^3+z^3=$**

(a) 90 (b) -90 (c) 0 (d) 368

21. **If $3x+\frac{1}{3x}=3$, then the value of $27x^3+\frac{1}{27x^3}$ is**

(a) -52 (b) 52 (c) 18 (d) -18

22. **If $\left(x+\frac{1}{x}\right)=4$, then $\left(x-\frac{1}{x}\right)$ is**

(a) $2\sqrt{2}$ (b) $\sqrt{6}$ (c) $2\sqrt{3}$ (d) $3\sqrt{2}$

23. **The factorization of $4a^2-4a+1$ is**

(a) $(2a-1)(2a+1)$ (b) $(2a-1)(1-2a)$

(c) $(2a+1)(2a+1)$ (d) $(2a-)(2a-1)$

24. **The factors of the expression $x^2 - \frac{y^2}{100}$ is**

(a) $\left(x-\frac{y}{10}\right)\left(x-\frac{y}{10}\right)$ (b) $\left(x+\frac{y}{10}\right)\left(x+\frac{y}{10}\right)$

(c) $\left(y+\frac{x}{10}\right)\left(y+\frac{x}{10}\right)$ (d) $\left(x+\frac{y}{10}\right)\left(x-\frac{y}{10}\right)$

25. **The value of $(-a+b+c)^2$ is**

(a) $a^2+b^2+c^2-2ab+2bc-2ca$ (b) $a^2-b^2-c^2-2ab+2bc-2ca$

(c) $x^2-y^2+z^2-2xy+3yz-4xz$ (d) $a^2+b^2-c^2-2ab-2bc-2ac$

26. **If $a+b+c=12$ and $a^2+b^2+c^2=50$, then the value of $ab+be+ca$, is**

(a) 44 (b) 22 (c) 23 (d) 47

27. **The factors of the expression x^4+x^2+1 is**

(a) $(x^2+1-x)(x^2-1+x)$ (b) $(x^2-1-x)(x^2-1-x)$

(c) $(x^2+1-x)(x^2-1-x)$ (d) $(x^2+1-x)(x^2+1+x)$

28. **The product of $\left(x-\frac{1}{x}\right)\left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}\right)\left(x^4+\frac{1}{x^4}\right)$ is**

(a) $\left(x^8-\frac{1}{x^8}\right)$ (b) $\left(x^4-\frac{1}{x^4}\right)$ (c) $\left(x^2-\frac{1}{x^2}\right)$ (d) $\left(x^8+\frac{1}{x^8}\right)$

29. **If $x+\frac{1}{x}=3$, then the value of $x^4+\frac{1}{x^4}$ is**

(a) 56 (b) 74 (c) 47 (d) 60

30. **If $x^2+\frac{1}{x^2}=123$. Then the value of $x^3-\frac{1}{x^3}$ is**

(a) 1340 (b) 1364 (c) 1358 (d) 1360

31. **The value of $\frac{(a^2-b^2)^3(b^2-c^2)^3+(c^2-a^2)^3}{(a-b)^3+(b-c)^3+(c-a)^3}$ is**

(a) $3(a+b)(b+c)(c+a)$ (b) $3(a-b)(b-c)(c-a)$

(c) $(a-b)(b-c)(c-a)$ (d) $(a+b)(b+c)(c+a)$

32. **If $p=2-a$, then the value of $a^3+6ap+p^3-8$ is**

(a) 1 (b) 0 (c) 3 (d) -1

33. **he factors of the expression $4x^2+4xy+y^2$ is**

(a) $(2x+y)(2x+y)$ (b) $(2x+y)(2x-y)$ (c) $(2x-y)(2x-y)$ (d) $(2x+x)(2y+x)$

34. **When the polynomial $f(x) = x^4 + 3x^3 - 2x^2 + x - 1$ is divided by $(x - 2)$ what will be the remainder?**

(a) 17 (b) 33 (c) 23 (d) 29

35. **If polynomials $2x^3 + ax^2 + 3x - 5$ and $x^3 + x^2 - 2x + a$ are divided by $(x - 2)$, the same remainders are obtained. Find the value of a.**

(a) -3 (b) 3 (c) -4 (d) -9

36. **For what value of k, $3x^4 + 2x^3 + 3kx^2 + 2x + 6$ is exactly divisible by $(x - 2)$?**

(a) 1 (b) 2 (c) -2 (d) $-\frac{8}{3}$

37. **What is the LCM of $x^2 - 1$, $x^2 - 4x + 3$ and $x^2 + 3x + 2$?**

(a) $(x+3)(x+1)$ (b) $(x^2-1)(x+2)(x-3)$

(c) $(x-1)(x-2)(x-3)$ (d) $(x^2-1)(x+2)(x+3)$

38. **What is the HCF of $x^2 - 5x + 6$, $x^2 - 7x + 12$ and $x^2 + 9x + 20$**

(a) 1 (b) $(x-3)$ (c) $(2x-5)$ (d) $x-4$

39. **If the LCM and HCF of two quadratic polynomials are $x^3 - 7x + 6$ and $(x - 1)$ respectively, find the polynomials.**

(a) $(x^2-3x+2),(x^2+2x+3)$ (b) $(x^2+3x-2),(x^2-2x+3)$

(c) $(x^2-3x+2),(x^2+2x-3)$ (d) $(x^2+3x+2),(x^2+2x+3)$

40. **If HCF and LCM of two terms a and b are x and y respectively and $a + b = x + y$, then $x^2 + y^2 = ?$**

(a) $a^2 - b^2$ (b) $2a^2 + b^2$ (c) $a^2 + b^2$ (d) $a^2 + 2b^2$

Answer Key & Hints

1. (c): $x=\left(\sqrt{2}+1\right)^{\frac{-1}{3}} \Rightarrow \frac{1}{x^3}=\sqrt{2}+1$

And $x^3=\frac{1}{\sqrt{2}+1}=\frac{1\left(\sqrt{2}-1\right)}{\left(\sqrt{2}+1\right)\left(\sqrt{2}-1\right)}=\sqrt{2}-1$

$\therefore x^3+\frac{1}{x^3} =\sqrt{2}-1+\sqrt{2}+1=2\sqrt{2}$

2. (a): $(3a+1)^2+(b-1)^2-(2c-3)^2=0$

$\Rightarrow 3a+1=0 \Rightarrow 3a=-1$

$\Rightarrow b-1=0 \Rightarrow b=1$

$\Rightarrow 2c-3=0$

$\therefore 2c=3$

$\therefore 3a+b+2c=-1+1+3=3$

3. (a): $x^2+y^2+z^2+\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}-6=0$

$\Rightarrow x^2+\frac{1}{x^2}-2+y^2+\frac{1}{y^2}-2+z^2+\frac{1}{z^2}-2=0$

$\Rightarrow \left(x-\frac{1}{x}\right)^2+\left(y-\frac{1}{y}\right)^2+\left(z-\frac{1}{z}\right)^2=0$

$\Rightarrow \left(x-\frac{1}{x}\right)=0 \therefore x^2-1=0 \Rightarrow x=\pm 1$

Similarly $y=\pm 1, z=\pm 1 \therefore x^2+y^2+z^2=1+1+1=3$

4. (c): $3\left[\frac{x^2}{yz}+\frac{y^2}{zx}+\frac{z^2}{xy}\right]=3\left[\frac{x^3+y^3+z^3}{xyz}\right]$

$=3\left[\frac{3xyz}{xyz}\right]=9$

5. (d): $x=3+2\sqrt{2}$

$xy=1 \Rightarrow y=\frac{1}{3+2\sqrt{2}}=\frac{1}{3+2\sqrt{2}}\times\frac{3-2\sqrt{2}}{3-2\sqrt{2}}$

$$= \frac{3-2\sqrt{2}}{9-8} = 3-2\sqrt{2}$$

$$\therefore x+y = 3+2\sqrt{2}+3-2\sqrt{2} = 6$$

$$\therefore \frac{x^2-3xy+y^2}{x^2+3xy+y^2} = \frac{(x+y)^2-5xy}{(x+y)^2+xy} = \frac{36-5}{36+1} = \frac{31}{37}$$

6. (b) $x = \sqrt{3}+\frac{1}{\sqrt{3}}$ *and* $y = \sqrt{3}-\frac{1}{\sqrt{3}}$

$$\therefore x+y = \sqrt{3}+\frac{1}{\sqrt{3}}+\sqrt{3}-\frac{1}{\sqrt{3}} = 2\sqrt{3}$$

And $xy - \left(\sqrt{3}+\frac{1}{\sqrt{3}}\right)\left(\sqrt{3}-\frac{1}{\sqrt{3}}\right) = \frac{8}{3}$

$$\therefore \frac{x^2}{y}+\frac{y^2}{x} = \frac{x^3+y^3}{xy} = \frac{(x+y)^3-3xy(x+y)}{xy}$$

$$= \frac{24\sqrt{3}-16\sqrt{3}}{\frac{8}{3}} = \frac{8\left(3\sqrt{3}-2\sqrt{3}\right)}{\frac{8}{3}} = 9\sqrt{3}-6\sqrt{3} = 3\sqrt{3}$$

7. (a): $a = \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}, b = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$ $\therefore a+b = \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}+\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} = \frac{\left(\sqrt{5}-\sqrt{3}\right)^2\left(\sqrt{5}+\sqrt{3}\right)^2}{\left(\sqrt{5}+\sqrt{3}\right)\times\left(\sqrt{5}-\sqrt{3}\right)} = \frac{2\left(\left(\sqrt{5}\right)^2+\left(\sqrt{3}\right)^2\right)}{5-3} = 5+3 = 8$

$$ab = \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} = 1$$

$$\therefore \frac{a^2-ab+b^2}{a^2+ab+b^2} = \frac{(a+b)^2-3ab}{(a+b)^2-3ab}$$

$$= \frac{8^2-1}{8^2-3} = \frac{63}{61}$$

8. (a): $a^4+b^4-x^2y^2 = 0$ .... (i)

We know,

$$a^6+b^6 = \left(a^2\right)^3+\left(b^2\right)^3 = \left(a^2+b^2\right)\left(a^4-x^2y^2+b^4\right) = \left(a^2+b^2\right)\times 0 = 0$$

9. (c): $xy(x-y) = 1 \Rightarrow x-y = \frac{1}{xy}$

Cubing both sides,

$$x^3-y^3-3xy(x-y) = \frac{1}{x^3y^3}$$

$$\Rightarrow x^3 - y^3 - 3xy \times \frac{1}{xy} = \frac{1}{x^3y^3}$$

$$\Rightarrow \frac{1}{x^3y^3} - x^3 + y^3 = 3$$

10. (a): $\left(a+\frac{1}{a}\right)^2 = 3 \Rightarrow a+\frac{1}{a} = \sqrt{3}$

On cubing both sides,

$$a^3 + \frac{1}{a^3} + 3\left(a+\frac{1}{a}\right) = 3\sqrt{3}$$

$$\Rightarrow a^3 + \frac{1}{a^3} = 3\sqrt{3} - 3\sqrt{3} = 0 \quad \Rightarrow a^6 + 1 = 0$$

$$\therefore a^{200} + a^{200} + a^{90} + a^{84} + a^{18} + a^{12} + a^6 + 1$$

$$= a^{200}\left(a^6+1\right) + a^{84}\left(a^6+1\right) + a^{12}\left(a^6+1\right) + \left(a^6+1\right) = 0$$

11. (a): $x + \frac{1}{x} = 3$

On squaring both sides,

$$x^2 + \frac{1}{x^2} + 2 = 9$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 9 - 2 = 7 \text{ ....(i)}$$

Expression

$$= \frac{x^4+3x^3+5x^2+3x+1}{x^4+1} = \frac{x^4+1+3x^3+3x+5x^2}{x^4+1} = \frac{x^2\left(x^2+\frac{1}{x^2}\right)+3x^2\left(x+\frac{1}{x}\right)+5x^2}{x^2\left(x^2+\frac{1}{x^2}\right)} = \frac{x^2\left(x^2+\frac{1}{x^2}\right)+3\left(x+\frac{1}{x}\right)}{x^2+\frac{1}{x^2}}$$

$$= \frac{7+3\times 3+5}{7} = \frac{21}{7} = 3$$

12. (d): $x + y + z = 6$

$$\Rightarrow x+y+z-6=0 \Rightarrow (x-1)+(y-2)+(z-3)=0 \therefore (x-1)^3+(y-2)^3+(z-3)^3$$

$$= 3(x-1)(y-2)(z-3)$$

13. (a): $x = 999$

Now, $\sqrt[3]{x(x^2+3x+3)+1} = \sqrt[3]{x^3+3x^2+3x+1}$

$x = 998$

14. (d): $\frac{x^4+x^5-x^8}{x^3}=x+x^2-x^5$

i.e., degree of the polynomial is 5.

15. (c) $p\left(\frac{1}{2}\right)=2.\left(\frac{1}{2}\right)^2=\frac{1}{2}+5=\frac{1}{2}-\frac{3}{2}+5=4$

16. **(d):** $p(2)=0\ \ k(2)^4-13(2)^3+k(2)^2+12\times2=0\Rightarrow16k-104+4k+24=0$

$\Rightarrow20k=80\Rightarrow k=4$

17. (b): $p(-1)=0\Rightarrow(-1)^4+3a(-1)+b=0$

$\Rightarrow+1-3a+b=0$

$\Rightarrow-3a+b=1$ ...,(1)

$p(1)=0\Rightarrow1^4+3a(1)+b=0\Rightarrow3a+b=-1$

.....(2)

Solving (1) and (2) we get $3+2b=-1$

18. (c): Area of rectangle $=3x^2+6xy+6y^2$

$=(3x+3y)(x+y)\therefore l=3x+3y$

19. (c) $\frac{x^3+9x^2+26x+24}{(x+2)(x+3)}=\frac{(x+2)(x+3)(x+4)}{(x+2)(x+3)}=(x+4)$

$\therefore$ 3rd factor $=(x+4)$

20. **(b):** $x+y+z=2+3-5=0$

$\therefore x+y+z=0\ \ \therefore x^3+y^3+z^3=3xyz$

$=3\times2\times3\times(-5)=-90$

21. (c): $\left(3x+\frac{1}{3x}\right)^3=27x^3+\frac{1}{27x^3}+3\left(3x+\frac{1}{3x}\right)$

$(3)^3=27x^3+\frac{1}{27x^3}+3\left(3x+\frac{1}{3x}\right)$

$\therefore27x^3+\frac{1}{27x^3}=27-9=18$

22. (c): $\left(x-\frac{1}{x}\right)^2=\left(x+\frac{1}{x}\right)^2-4=16-4=12$

$\therefore x-\frac{1}{x}=2\sqrt{3}$

23. **(d):** $4a^2-4a+1=(2a)^2-2.2a.1+1^2$

$=(\mathbf{2a}-1)^{\mathbf{2}}=(\mathbf{2a}-1)(\mathbf{2a}-1)$

24. (d): $x^2-\frac{1}{y^2}=x^2-\left(\frac{y}{10}\right)^2=\left(x+\frac{y}{10}\right)\left(x-\frac{y}{10}\right)$

25. (a): $(-a+b+c)^2=a^2+b^2+c^2-2ab+2bc-2ac$

26. (d): $(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ca)$

$144=50+2(ab+bc+ca)$

$\therefore ab+bc+ca=\frac{94}{2}=47$

27. (d): x^4+x^2+1

$x^4+x^2+1=x^4+x^2+1+x^2-x^2$

$=(x^4+2x^2+1)-x^2$

$=(x^2+1)^2-x^2=(x^2+1-x)(x^2+1+x)$

28. (a):We have,

$\left(x-\frac{1}{x}\right)\left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}\right)\left(x^4+\frac{1}{x^4}\right)$

$=\left(x^2-\frac{1}{x^2}\right)\left(x^2+\frac{1}{x^2}\right)\left(x^4+\frac{1}{x^4}\right)$

$=\left\{\left(x^2\right)^2-\left(\frac{1}{x^2}\right)^2\right\}\left(x^4+\frac{1}{x^4}\right)$

29. (c): Since $\left(x+\frac{1}{x}\right)=3,$, then

$x^2+\frac{1}{x^2}=7 \Rightarrow x^4+\frac{1}{x^4}=47.$

30. (b): We know that

$$\left(x-\frac{1}{x}\right)^2 = x^2+\frac{1}{x^2}-2 \Rightarrow \left(x-\frac{1}{x}\right)^2 = 123-2 \Rightarrow \left(x-\frac{1}{x}\right)^2 = 121$$

$$\left(x-\frac{1}{x}\right)^2 = 11^2 \Rightarrow x-\frac{1}{x} = 11 \ \Rightarrow \left(x+\frac{1}{x}\right) = 11^3 \Rightarrow x^3-\frac{1}{x^3}-3\left(x-\frac{1}{x}\right) = 1331$$

31. (d): We have

$$\left(a^2-b^2\right)+\left(b^2-c^2\right)-\left(c^2-a^2\right)=0$$

$$\therefore (a^2-b^2)^3+(b^2-c^2)^3+(c^2-a^2)^3$$

$$=(a^2-b^2)(b^2-c^2)(c^2-a^2)$$

$$=3(a-b)(a+b)(b-c)(b+c)(c-a)(c+a)$$

Similarly, we have

$$(a-b)+(b-c)+(c-a)=0$$

$$\Rightarrow (a-b)^3+(b-c)^3+(c-a)^3$$

$$=3(a-b)(b-c)(c-a)$$

And proceed.

32. (b): We have,

$$P=2-a \Rightarrow a+p-2=0$$

Now, $a^3+6ap+p^3-8=a^3+p^3+(-2)^3-3ap(-2)$

$$=\{a+p+(-2)\}\{a^2+p^2+(-2)^2-ap-p(-2)-a(-2)\}=\{a+p-2\}\{a^2+p^2+4-ap+2p+2a\}$$

$$=0\times\left(a^2+p^2+4-ap+2p+2a\right)=0.$$

33. (a): $4x^2+4xy+y^2=4x^2+2xy+2xy+y^2$

$$=2x(2x+y)+y(2x+y)=(2x+y)(2x+y)$$

34. (b): $f(2)=2^4+3\times2^3-2\times2^2+2-7$

$$=16+24-8+2-1-19 \therefore Remainder=33$$

35. (a):

$$f(x)^2=2x^3+ax^2+3x-5;$$

$$g(x)=x^3+x^2-2x+a$$

By Remainder Theorem,

$f(2)=(2\times 2^3+a\times 2^2+3\times 2-5)=17+4a$

Again, $g(2)=(2^3+2^2-2\times 2+a)=8 \therefore 17+4a=8+a \Rightarrow 3a=-9 \qquad \Rightarrow a=-3$

36. (d): Here, $x-2=0 \Rightarrow x=2$

By Factor Theorem, $f(2)=0$

$\Rightarrow 3(-2)^4+2(-2)^3+3k(-2)^2+(-2)+6=0$

$\Rightarrow 48-16+12k-6+6 \Rightarrow 12k=-32$

$\Rightarrow k=\frac{-8}{3}$

37. (b) (i) $x^2-1=x^2-(-1)^2=(x-1)(x+1)$

(ii) $x^2-4x+3=x^2-3x-x+3$

$=x(x-3)-1(x-3)=(x-3)(x-1)$

(iii) $x^2+3x+2=x^2+2x+x+2 \ =x(x+2)+1(x+2)=(x+2)(x+1)$

$LCM=(x-1)(x+1)(x+2)(x-3)$

38. (a) $(x^2-5x+6)(x^2-7x+12) and\ x^2+9x+20 \ \ x^2-5x+6=(x-2)(x-3)$

$x^2-7x+12=(x-3)(x-4)$

$x^2+9x+20=(x+4)(x+5)$

Hence HCF = 1 as no term is common.

39. (c): Putting $x-1=0$ *i.e.*, $x=1$ *in* $(x-1)$

Remainder $=(+1)^3-7(1)+6=1-7+6=0$

$\therefore (x-1)$ is a factor of expression

$x^3-7x+6=0$

Now, $x^3-7x+6=x^2(x-1)+x(x-1)-6(x-1)$

$=(x-1)(x^2+x-6)$

$=(x-1)(x-2)(x-3)$

$=(x-1)[x^2+3x-2x-6]$

$=(x-1)[x(x+3)-2(x+3)]$

$=(x-1)(x-2)(x-3)$

LCM $= x^3-7x+6=(x-1)(x-2)(x-3)$ and their $HCF=(x-1)$

$\therefore (x-1)$ is common in both

$\therefore$ First expression $=(x-1)(x-2)$

$=x^2-3x+2$ and second expression

$=(x-1)(x+3)=x^2+2x-3$

40. (c): $\because p(x)\times q(x)=LCM\times HCF$ And, proceed.